Work and Time

~ Quantitative Aptitude · SSC Exams ~

← circled = key info!

The whole chapter rests on one idea: turn every "days" into a rate of work per day, add or subtract the rates, then convert back to time. Master the **LCM method** and most sums become mental maths.

① The Basic Idea — one-day work

Treat the whole job as **1 unit of work**. If a person needs **n** days for the whole job, then in one day he does the n^{th} part of it.

Formula — time ↔ one-day work

- Work in **n** days ⇒ one-day work = **1/n**
- One-day work = **1/n** ⇒ whole work takes **n** days

∴ time and one-day work are just reciprocals of each other.

Worked example — A finishes in 5 days

Given — A completes the whole job in **5** days.
One-day work → A does **1/5** of the job each day.
In 3 days → 3 × 1/5 = **3/5** done ∴ 2/5 is still left. ✅

Worked example — reverse direction

Given — B's one-day work = **1/8**.
Whole job → reciprocal → B needs **8 days** for the full work. ✅

Try these

10 days → one-day = **1/10**

6 days → in 2 days does **1/3**

one-day 1/12 → **12 days**

one-day 1/20 → **20 days**

Golden habit → the moment you read "does it in **n** days", write **1/n** beside the name. Every later step is add / subtract of these fractions.

② Working Together — two people

When two people work at the same time, their **one-day works add up**. Convert each to a fraction, add, then flip to get the combined time.

Formula — A in a days, B in b days

- Together's one-day work = **$1/a + 1/b$**
- Time working together = **$ab/(a+b)$** (product ÷ sum)

∴ two together are always **faster** than either one alone.

Worked example — A: 10 days, B: 15 days

Step 1 — one-day works: $A = 1/10$, $B = 1/15$.
Step 2 — add: $1/10 + 1/15 = 3/30 + 2/30 = \mathbf{5/30 = 1/6}$.
Step 3 — flip → together they finish in **6 days**.
Check → $ab/(a+b) = (10×15)/(10+15) = 150/25 = \mathbf{6}$. ✅

Worked example — A: 12 days, B: 6 days

Together = $ab/(a+b) = (12×6)/(12+6) = 72/18 =$ **4 days**.
Check → $1/12 + 1/6 = 1/12 + 2/12 = 3/12 = 1/4$ → 4 days. ✓

The **ab/(a+b)** shortcut works for **exactly two** workers. For three or more, add the fractions instead (next page).

Try these (use product ÷ sum)

20 & 30 → 600/50 = **12 d**
15 & 10 → 150/25 = **6 d**
6 & 12 → 72/18 = **4 d**
8 & 8 → 64/16 = **4 d**
9 & 18 → 162/27 = **6 d**
5 & 20 → 100/25 = **4 d**

✓ Equal workers → if both take the same **a** days, together = **a/2**. Two men each 8 d finish in 4 d.

Working Together — three people

Formula — A, B, C together

- One-day work = **$1/a + 1/b + 1/c$**
- Time = reciprocal of that sum

∴ add fractions over the LCM of the denominators, then flip.

Worked example — A: 10, B: 15, C: 30

Step 1 — LCM of 10, 15, 30 = **30**.
Step 2 — $1/10 + 1/15 + 1/30 = 3/30 + 2/30 + 1/30 = \mathbf{6/30 = 1/5}$.
Answer → together they finish in **5 days**. ✅

Worked example — A: 6, B: 8, C: 12

LCM of 6, 8, 12 = **24**: $1/6 + 1/8 + 1/12 = 4/24 + 3/24 + 2/24 = \mathbf{9/24 = 3/8}$.
Time → $8/3 = $ **$2/3$ i.e. 2 days 16 hours** ($2/3 \times 24$ h). ✅

Finding the missing man

If the **together** rate and one person's rate are known, subtract to get the third.

Worked — A+B do it in 10 d, A alone in 15 d. B?

Together = $1/10$, **A** = $1/15$.
B = $1/10 - 1/15 = 3/30 - 2/30 = 1/30$ → **B takes 30 days**. ✅

"Together minus one = the other." Always keep the together-rate and each person's rate over the **same denominator** so you can just subtract the tops.

Try these

4, 6, 12 → 3+2+1=6/12 → **2 d**

5, 10, 10 → 2+1+1=4/10 → **2.5 d**

A+B 12, A 20 → B = **30 d**

A+B 8, B 24 → A = **12 d**

③ Efficiency ∝ 1/Time

"Efficiency" is just **work done per day**. A faster worker does more per day, so he needs **less** time. Hence efficiency and time are **inversely** related.

Rule — efficiency vs time

- Efficiency ∝ 1/Time (more efficient → fewer days)
- If efficiencies are in ratio **x : y**, then times are in ratio **y : x** (flip it)

∴ "A is twice as efficient as B" ⇒ A takes **half** the time of B.

Worked example — A twice as efficient as B

Given — B alone finishes in **12** days.

A's time → half of B = **6 days**.

Together → $1/6 + 1/12 = 2/12 + 1/12 = 3/12 = 1/4$ → **4 days**. ✅

Worked example — ratio flip

Efficiencies A : B = **3 : 2** ⇒ times A : B = **2 : 3**.

If A takes **10** days, B takes $10 \times 3/2$ = **15 days**. ✅

Percentage-more efficiency

A is 25% more efficient than B; B takes 25 d. A?

A's efficiency = $1.25 \times B$ ⇒ A's time = $25 \div 1.25$ = **20 days**.

25% more efficient = efficiency ratio 5 : 4 → time ratio 4 : 5 → $25 \times 4/5 = 20$. ✓

Never flip the wrong way! **More efficient → fewer** days. If A's efficiency is bigger, A's day-count must be smaller.

Try these

A 3× B, B 30 d → A = **10 d**

A half as fast as B, B 8 d → A = **16 d**

Eff 4:5, A 20 d → B = **16 d**

Eff 2:3, B 10 d → A = **15 d**

④ The LCM Method — the fastest way ☆

Instead of adding awkward fractions, make the whole work a **convenient number of units** — the **LCM** of all the given days. Then everyone's rate is a clean whole number.

The 4-step LCM recipe

- **Total work** = LCM of the given day-figures (units)
- **Each rate** = Total work ÷ that person's days (units/day)
- **Combined rate** = add the individual rates
- **Time** = Total work ÷ combined rate

∴ no fractions, no LCM of denominators — just divide and add.

Worked example — A: 12, B: 15, C: 20 days

Step 1 — Total = LCM(12, 15, 20) = **60 units**.
Step 2 — rates: A = 60/12 = **5**, B = 60/15 = **4**, C = 60/20 = **3** units/day.
Step 3 — combined = 5 + 4 + 3 = **12** units/day.
Step 4 — Time = 60 ÷ 12 = **5 days**. ✅

Worked — A: 10, B: 15 days (two workers)

Total = LCM(10,15) = **30**; A = 3, B = 2 → combined 5 → 30/5 = **6 days**.
Same answer as the fraction method — but faster to write. ✓

✓ Think of the work as a wall of **LCM bricks**. Each worker lays a fixed number of bricks a day — a picture that makes "leaves after k days" and "alternate days" trivial.

LCM Method — more worked examples

Worked — A: 20, B: 30, C: 60 days

Total = LCM = **60**; rates A = 3, B = 2, C = 1 → combined **6** → 60 / 6 = **10 days**. ✅

Worked — A: 8, B: 12 days

Total = LCM = **24**; A = 3, B = 2 → combined **5** → 24 / 5 = **4.8 days** (4 days 19.2 h). ✅

Worked — find the third worker by units

Given — A+B+C finish in 6 d; A in 12 d, B in 18 d. Find C.

Total = LCM(6,12,18) = **36**; A = 3, B = 2, all-three = 6.

C's rate = 6 − 3 − 2 = **1** → C alone = 36 / 1 = **36 days**. ✅

Efficiency ratio straight from the units

In the top example the rates were **5 : 4 : 3** — that **is** the efficiency ratio of A : B : C. The LCM method hands you the efficiencies for free.

Try these (LCM in your head)

Days	LCM	Rates & sum	Together
6, 12	12	2 + 1 = 3	**4 d**
15, 10	30	2 + 3 = 5	**6 d**
10, 12, 15	60	6 + 5 + 4 = 15	**4 d**
16, 24	48	3 + 2 = 5	**9.6 d**
9, 18, 6	18	2 + 1 + 3 = 6	**3 d**

If the days don't share nice factors, the LCM is just their product — e.g. 7 & 11 → total 77 units. The method still works.

⑤ Someone Leaves / Joins Part-way

*Use LCM units. Count the units done **while everyone is present**, subtract from the total, then let the remaining worker(s) finish the **leftover** at their rate.*

Plan of attack

- Work done in first phase = (sum of present rates) × days
- Leftover = Total − work done so far
- Extra time = Leftover ÷ (rate of whoever keeps working)

Worked — A: 10, B: 15; A leaves after 2 days

Total = LCM = 30; A = 3, B = 2 units/ day.

First 2 days (both) → (3+2)×2 = **10 units** done.

Leftover = 30 − 10 = 20; B alone → 20 ÷ 2 = **10 days**.

Total time = 2 + 10 = **12 days**. ✅

Worked — A starts alone, B joins later

Given — A: 12 d, B: 12 d. A works alone 3 d, then B joins.

Total = 12; A = 1, B = 1. First 3 d → 3 units. Leftover = 9.

Together rate 2 → 9 ÷ 2 = 4.5 d → total = 3 + 4.5 = **7.5 days**. ✅

Worked — B leaves in the middle

Given — A: 10, B: 15; both start, B leaves after 3 d, A finishes.

Total 30; A = 3, B = 2. First 3 d → (5)×3 = 15 units. Leftover 15.

A alone → 15 ÷ 3 = 5 d → total = 3 + 5 = **8 days**. ✅

*Read carefully — is the question asking for the **total** time or only the **extra** days after someone left? Answer exactly what is asked.*

⑥ Alternate-Days Working ⇄

Here workers take turns — one works day 1, the other day 2, and so on. Add up **one full cycle** of turns, see how many cycles fit, then finish the small remainder.

Cycle method

- 1 cycle = 2 days (A's day + B's day) = sum of their day-units
- No. of full cycles = Total ÷ cycle-units (take the whole part)
- Finish the leftover with whoever's turn is next

Worked — A: 4 d, B: 6 d, alternate (A starts)

Total = LCM(4,6) = 12; A = 3, B = 2 units/day.

1 cycle (A then B) = 3 + 2 = **5 units** in 2 days.

2 cycles = 10 units in 4 days; leftover = 12 − 10 = **2 units**.

Day 5 is A's turn (rate 3): needs 2/3 of a day for 2 units.

Total time = 4 + 2/3 = **$4\frac{2}{3}$ days** (4 days 16 h). ✅

Worked — A: 6, B: 8, alternate (A starts)

Total = LCM = 24; A = 4, B = 3. Cycle = 7 units/2 days.

3 cycles = 21 units in 6 days; leftover 3. Day 7 is A (rate 4) → 3/4 day.

Total = 6 + 3/4 = **$6\frac{3}{4}$ days**. ✅

Who starts **matters**! Swapping the first worker can change the answer, because the leftover is finished at a different rate. Always follow the given order.

✓ Don't stop at the last full cycle — check if the **next single day** already finishes the job before splitting into a fraction of a day.

⑦ The M · D · H Chain Rule

When the **number of men**, **days** and **hours per day** all change, use the master proportion. It keeps "work per man-hour" constant.

Formula — men, days, hours, work

$$M_1 \cdot D_1 \cdot H_1 \,/\, W_1 \;=\; M_2 \cdot D_2 \cdot H_2 \,/\, W_2$$

M = men · D = days · H = hours/day · W = amount of work

If the job is the **same** ($W_1 = W_2$), it shortens to $M_1D_1H_1 = M_2D_2H_2$.

Worked — change men and hours

Given — 10 men build a wall in 8 days at 6 h/day. How many days will 12 men need at 8 h/day (same wall)?

Set up → $10 \times 8 \times 6 = 12 \times D_2 \times 8$.

Solve → $480 = 96 \cdot D_2 \rightarrow D_2 =$ **5 days**. ✅

Worked — how many men?

12 men finish in 10 days. To finish in 8 days: $12 \times 10 = M_2 \times 8 \rightarrow M_2 =$ **15 men**. ✅

Worked — with different work amounts

10 men in 6 days dig a trench. How many days for 8 men to dig a trench **twice** as long?

$(10 \times 6)/1 = (8 \times D)/2 \rightarrow 60 = 4D \rightarrow D =$ **15 days**. ✅

Keep every quantity on the correct side. More work → W goes **up**, so days go up. Fewer men → days go up. The formula balances it automatically.

⑧ Work & Wages

Rule – pay follows work done

- Wages are shared in the ratio of **work done** = ratio of (efficiency × time).
- If all work for the **same time**, split in the ratio of their **rates** — i.e. 1/time each.

Worked – two workers share ₹3000

Given — A: 10 d, B: 15 d; they finish together for ₹3000.

Ratio = 1/10 : 1/15 = 3 : 2 (multiply by 30) → 5 parts.

Shares → A = 3/5 × 3000 = **₹1800**, B = 2/5 × 3000 = **₹1200**. ✅

Worked – three workers share ₹1800

Given — A: 6, B: 8, C: 12 days, together for ₹1800.

Ratio = 1/6 : 1/8 : 1/12 → ×24 → 4 : 3 : 2 (9 parts).

Shares → A = 4/9×1800 = **₹800**, B = **₹600**, C = **₹400**. ✅

Worked – unequal working times

A works 5 days, B works 3 days on a ₹800 job; rates are equal. Share = 5 : 3 → A = **₹500**, B = **₹300** (pay by work actually done).

Pay is **never** in the ratio of days-taken. It follows **work done** – which for equal time means the ratio of **efficiencies (1/time)**.

⑨ Fraction of Work & "How Much is Left"

Key relations

- Work done in **t** days = t × (one-day work) = **t / n**.
- Work left = **1 − t / n**.
- If a fraction **f** is done in t days → whole = **t / f** days.

Worked — how much is left?

A can do a job in 15 days and works for 6 days.

Done = 6 / 15 = **2 / 5** → Left = 1 − 2 / 5 = **3 / 5** of the work. ✅

Worked — find the whole from a fraction

A does **1 / 3** of a job in 5 days. Whole job? → 5 ÷ (1 / 3) = **15 days**.

Remaining 2 / 3 → 15 × 2 / 3 = **10 more days**. ✅

A peek ahead — Pipes & Cisterns

A **filling pipe** is a worker doing **+** work; an **emptying pipe / leak** does **−** work. Add the rates with their signs — same maths, one may be negative.

Worked — fill vs empty

Inlet fills a tank in 6 h; outlet empties it in 8 h. Both open?

Net = 1 / 6 − 1 / 8 = 4 / 24 − 3 / 24 = **1 / 24** → tank fills in **24 hours**. ✅

✓ *If the empty-rate is **bigger** than the fill-rate, the net is negative — the tank never fills. Watch the sign before you flip.*

Every Work-and-Time trick carries over to Pipes & Cisterns — just relabel "days" as "hours" and allow negative rates.

Special Cases & Handy Shortcuts

Men ↔ Women equivalence

6 men **or** 8 women finish a job in 24 days. Then 6M = 8W in output → 8W = 6M.
6 men + 8 women = 6M + 6M = **12 men**. 6 men take 24 d → 12 men take **12 days**. ✅

Together & one alone → the other

A + B in 4 d, A alone in 6 d. LCM 12: together = 3/day, A = 2/day.
B = 3 − 2 = 1 → B alone = 12/1 = **12 days**. ✅

"Twice as efficient" pair

A is twice as good as B; together they finish in 8 d. A alone?
Rates A : B = 2 : 1 → together 3 parts/day. Total = 3×8 = 24 parts.
A alone = 24 ÷ 2 = **12 days**; B alone = 24 ÷ 1 = **24 days**. ✅

Pocket shortcuts

- Two workers a, b days → together = **ab/(a+b)**.
- A + B = x d, A = a d → B = **ax/(a−x)** days.
- Efficiency ratio = reverse of the time ratio.
- Wages ratio = ratio of work done = efficiency × time.

Check the B = ax/(a−x) shortcut

A + B = 4 d, A = 6 d → B = (6×4)/(6−4) = 24/2 = **12 days** — matches above. ✓

These shortcuts are just the LCM method in disguise. When unsure, fall back to **units** — it never lies.

More Solved Examples

1. A: 15 d, B: 20 d – together?
LCM 60: A = 4, B = 3 → sum 7 → 60/7 = **$8\frac{7}{8}$ days** (8 4/7).

2. A twice as efficient as B; together 8 d. A alone?
Rates 2 : 1 → total = 3×8 = 24 units → A = 24/2 = **12 days**.

3. A: 10, B: 12, C: 15 – together?
LCM 60: 6 + 5 + 4 = 15 → 60/15 = **4 days**.

4. A: 12, B: 16; both start, A leaves after 3 d, B finishes.
LCM 48: A = 4, B = 3. First 3 d → (7)×3 = 21; left 27. B: 27/3 = 9 d → total = **12 days**.

5. 6 men, 16 d, 9 h/day. How many days for 8 men at 12 h/day?
6×16×9 = 8×D×12 → 864 = 96D → D = **9 days**.

6. A: 20, B: 30; share ₹500 (worked together).
Ratio 1/20 : 1/30 = 3 : 2 → A = **₹300**, B = **₹200**.

7. A: 5, B: 10; alternate days, A starts. Time?
LCM 10: A = 2, B = 1. Cycle = 3/2 d. 3 cycles = 9 in 6 d; left 1; day 7 (A) → 1/2 d → **6½ days**.

8. A does 2/5 of a job in 8 d. Time for the rest?
Whole = 8 ÷ (2/5) = 20 d; remaining 3/5 → 20×3/5 = **12 days**.

More Solved Examples — cont.

9. A + B in 4 d, A alone in 6 d. B alone?
LCM 12: together 3, A = 2 → B = 1 → B alone = **12 days**.

10. Inlet fills in 10 h, leak empties in 15 h. Both open?
Net = 1/10 − 1/15 = 3/30 − 2/30 = 1/30 → fills in **30 hours**.

11. A: 25, B: 20; A is how much % less efficient than B?
Rates A = 4, B = 5 (LCM 100). A is 1/5 = **20% less** efficient than B.

12. A: 18, B: 9, C: 6 — all three together?
LCM 18: 1 + 2 + 3 = 6 → 18/6 = **3 days**.

13. A: 30, B: 40; start together, B leaves after 10 d. A finishes rest.
LCM 120: A = 4, B = 3. 10 d → (7)×10 = 70; left 50; A: 50/4 = 12.5 d → total = **22.5 days**.

14. 16 men finish in 15 d. After 6 d, 8 more men join. Remaining days?
Total = 16×15 = 240 man-days. In 6 d, 16×6 = 96 done; left 144. Now 24 men → 144/24 = **6 days**.

15. A: 24, B: 6, C: 12; wages ₹1400 (together).
Ratio 1/24 : 1/6 : 1/12 = 1 : 4 : 2 (7 parts). A = **₹200**, B = **₹800**, C = **₹400**.

✓ *Notice how "man-days" (Q14) is just the LCM method with total = M × D. Same idea, different label.*

Higher-Level Solved Examples ☆

1. A: 10, B: 15; both start, A leaves, B works 5 more days to finish. When did A leave & total time?

LCM 30: A = 3, B = 2. B's last 5 d alone = 2×5 = 10 units. Remaining 20 done together at 5/day → 20/5 = 4 d. **A left after 4 days; total = 9 days**.

2. Two pipes fill in 12 h & 15 h; an outlet empties in 20 h. All open – time to fill?

LCM 60: +5 +4 −3 = +6 → 60/6 = **10 hours**.

3. 6 men or 8 women finish in 24 d. How long for 6 men + 8 women together?

8W = 6M → 6M + 8W = 12M. 6M take 24 d → 12M take **12 days**.

4. A is 25% more efficient than B. Together they take 20 d. B alone?

Eff A : B = 5 : 4 → together 9 parts/day. Total = 9×20 = 180 parts. B = 180/4 = **45 days**.

5. A: 6, B: 8; work on alternate days, B starts first. Time?

LCM 24: A = 4, B = 3. Cycle (B then A) = 7/2 d. 3 cycles = 21 in 6 d; left 3; day 7 is B (rate 3) → 1 full day. **Total = 7 days**.

6. A & B do a job in 12 d for ₹6000. With C's help they finish in 8 d. C's share?

Together (A+B) = 1/12, all three = 1/8. C = 1/8 − 1/12 = 1/24. Ratio (A+B) : C = 1/12 : 1/24 = 2 : 1 → C = 1/3 × 6000 = **₹2000**.

Example 5 vs page 8: **same numbers, different starter** → 7 days here but 6¾ when A starts. Order changes the leftover!

Practice Set

1. A does it in 8 d — one-day work?
2. A: 12, B: 6 — together?
3. A: 10, B: 15, C: 30 — together?
4. A twice as good as B, B: 18 d — A?
5. LCM: A: 12, B: 15, C: 20 — together?
6. A: 20, B: 30; A leaves after 6 d — B's extra days?
7. A: 4, B: 6; alternate, A starts — time?
8. 10 men, 8 d, 6 h/d → 12 men, 8 h/d — days?
9. A: 10, B: 15; share ₹500 — each?
10. A does 3/5 in 9 d — whole job?
11. A+B in 6 d, A in 9 d — B alone?
12. Fill 6 h, empty 8 h — net time?
13. A: 10, B: 12, C: 15 — together?
14. 6 men or 8 women in 24 d — 6M+8W?

Answer Key — with working

1→ 1/8 · **2→ 4 d** (LCM12: 1+2=3) · **3→ 5 d** (LCM30: 3+2+1=6) · **4→ 9 d** (half of 18) · **5→ 5 d** (LCM60: 5+4+3=12) · **6→ 15 d** (LCM60: 6×5=30 done, left 30, B 2/d) · **7→ 4⅔ d** (LCM12: 2 cyc=10 in 4 d, A does last 2/3 d) · **8→ 5 d** (10×8×6=12×D×8) · **9→ ₹300 & ₹200** (3:2) · **10→ 15 d** (9÷3/5) · **11→ 18 d** (LCM18: 3−2=1) · **12→ 24 h** (1/6−1/8=1/24) · **13→ 4 d** (LCM60: 6+5+4=15) · **14→ 12 d** (8W=6M → 12M).

Quick Revision

✓ n days → one-day work 1/n. Two together = ab/(a+b); three → add 1/a+1/b+1/c.
✓ **LCM method**: total = LCM of days; rate = total/days; time = total/(sum of rates).
✓ Efficiency ∝ 1/time; $M_1D_1H_1/W_1 = M_2D_2H_2/W_2$; wages ∝ work done.

☆ Rate × time — finish the work! ☆