Number System

~ Quantitative Aptitude · SSC Exams ~

← circled = key info!

The whole subject stands on numbers. Master how they are classified and how their **factors, digits & remainders** behave.

① Classification of Numbers

Real Numbers (R)

↓

Rational (Q)

p/q, $q \neq 0$ — terminating or recurring decimals.

$3/4 = 0.75$ · $22/7$ · $0.333...$

holds → $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q}$

Irrational

non-terminating & non-recurring decimals.

$\sqrt{2}$ · $\sqrt{3}$ · π · e

cannot be written as p/q.

Natural (N): 1, 2, 3, 4... (counting)

Whole (W): 0, 1, 2, 3... (N + zero)

Integers (Z): ...−2,−1,0,1,2...

Imaginary: $i = \sqrt{-1}$

Complex number = $a + bi$ (a, b real). Every real number is complex with $b = 0$. Imaginary unit i satisfies $i^2 = -1$.

Even & Odd

Even → divisible by 2. 0, 2, 4, 6...

Odd → not divisible by 2. 1, 3, 5, 7...

Perfect Squares & Cubes — memorise!

- **Squares:** 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225...
- **Cubes:** 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000...

Smallest **natural** = 1, smallest **whole** = 0 ∴ **0 is whole but not natural.** Every natural → whole → integer → rational → real.

Primes, Composites & Special numbers

Prime → exactly **two** factors (1 and itself). 2, 3, 5, 7, 11...

Composite → **more than two** factors. 4, 6, 8, 9, 10...

Prime facts
- **2** is the only **even** prime.
- Smallest prime = **2**.
- There are **25** primes below 100.

Composite facts
- Smallest composite = **4**.
- **1** is neither prime nor composite.
- Every composite → product of primes.

Co-prime → two numbers whose **HCF = 1** (need not be primes). 8 & 15, 9 & 16

Twin primes → a pair of primes differing by **2**. (3,5) (5,7) (11,13)

Perfect number → sum of proper factors = the number itself. 6 = 1+2+3 · 28 = 1+2+4+7+14

Co-prime ≠ prime! **8 & 15** are co-prime although each is composite — only their **common factor** is 1.

② Face value vs Place value

Face value = the digit itself (never changes).

Place value = digit × value of its position.

Example — in the number **3752**, look at the digit **7**:
- Face value of 7 = **7**
- 7 sits in the **hundreds** place ∴ Place value = 7 × 100 = **700**

③ Even & Odd Operations

E ± E = **E**	E × E = **E**
O ± O = **E**	O × O = **O**
E ± O = **O**	E × O = **E**

Product is ODD only when BOTH are odd

Primes below 50 — learn by heart

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 (15 primes · **25** primes below 100)

The **product** of two consecutive integers $n(n+1)$ is always **even**; the **sum** of two consecutive integers is always **odd**.

④ Divisibility Rules

By	Rule	Example
2	last digit is even (0,2,4,6,8)	34 ✓
3	digit-sum divisible by 3	1+2+3=6 ✓
4	last **two** digits divisible by 4	73**16** ✓
5	ends in 0 or 5	27**5** ✓
6	divisible by 2 **and** 3	114 ✓
7	double last digit, subtract from rest; repeat	161→16−2=14 ✓
8	last **three** digits divisible by 8	7**512** ✓
9	digit-sum divisible by 9	9+8+1=18 ✓
10	ends in 0	45**0** ✓
11	(Σ odd-place digits) − (Σ even-place digits) = 0 or a multiple of 11	121: (1+1)−2=0 ✓
13	multiply last digit by 4, add to rest; repeat	143→14+12=26 ✓

✓ For a **composite** divisor, split it into **co-prime** factors and test each: 12 = 3 × 4 · 15 = 3 × 5 · 72 = 8 × 9.

⑤ Testing for a Prime

To check if **n** is prime, test divisibility by every prime **≤ √n** only. If none divides it → prime.

Worked example — is 197 prime?

Step 1 — √197 ≈ 14, so test primes ≤ 14: **2, 3, 5, 7, 11, 13**.

Step 2 — 197 is odd (≠2); 1+9+7=17 (≠3); doesn't end 0/5 (≠5).

Step 3 — 197÷7 → rem 1; ÷11 → rem 10; ÷13 → rem 2.

Answer → no prime ≤√197 divides it ∴ **197 is prime**. ✅

More divisibility shortcuts

25 → last two digits are 00, 25, 50 or 75

125 → last three digits ÷ 125

12 → divisible by 3 **and** 4

Worked — is 48516 divisible by 11?

Digits from the right: 6, 1, 5, 8, 4. Σ odd places = 6+5+4 = 15; Σ even places = 1+8 = 9.
Difference = 15 − 9 = **6** → not 0 or a multiple of 11 ∴ **not** divisible by 11.

⑥ Unit Digit – Cyclicity

The last digit of a power repeats in a **cycle**. Take the **exponent ÷ cycle-length**; the remainder points to the term.

Base ends in	Cycle of unit digits	Length
0, 1, 5, 6	digit stays the same	1
4	4, 6	2
9	9, 1	2
2	2, 4, 8, 6	4
3	3, 9, 7, 1	4
7	7, 9, 3, 1	4
8	8, 4, 2, 6	4

Worked example — unit digit of 7^{84}

Step 1 — base ends in 7 → cycle (7, 9, 3, 1), length **4**.

Step 2 — 84 ÷ 4 → remainder **0**. Remainder 0 → take the **last** term of the cycle.

Answer → last term = **1** ∴ unit digit of 7^{84} is **1**. ✅

Shortcut → for length-4 cycles, only the **last two digits** of the exponent matter (divide them by 4). Remainder 1,2,3,0 → 1st, 2nd, 3rd, 4th term.

Quick check

3^{65} → 65÷4 rem 1 → **3**

2^{54} → 54÷4 rem 2 → **4**

9^{27} → 27 odd → **9**

4^{50} → 50 even → **6**

More worked — 2^{100} and 3^{47}

- 2^{100}: cycle (2, 4, 8, 6) length 4; 100 ÷ 4 rem 0 → last term → **6**.
- 3^{47}: cycle (3, 9, 7, 1) length 4; 47 ÷ 4 rem 3 → 3rd term → **7**.

Unit digit of a perfect square

A square can end only in **0, 1, 4, 5, 6, 9** — **never** in 2, 3, 7 or 8. Unit digit of a product = unit digit of (product of the unit digits).

⑦ Number of Factors

First write N in prime-factor form: $N = a^p \cdot b^q \cdot c^r$ (a, b, c distinct primes).

Key formulas — $N = a^p \cdot b^q \cdot c^r$

- Number of factors = $(p+1)(q+1)(r+1)$
- Sum of factors = $(a^{p+1}-1)/(a-1) \cdot (b^{q+1}-1)/(b-1) \cdot \ldots$
- Product of factors = $N^{(\text{no. of factors})/2}$
- Ways to write N = product of two factors = **(no. of factors)/2**

  ∴ if N is a perfect square → use **(no. of factors + 1)/2**

Worked example — N = 72

Step 1 — $72 = 8 \times 9 = 2^3 \cdot 3^2$ ∴ $p = 3$, $q = 2$.

Factors → $(3+1)(2+1) = 4 \times 3 =$ **12 factors**.

Sum → $(2^4-1)/(2-1) \cdot (3^3-1)/(3-1) = 15 \times 13 =$ **195**.

Product → $72^{12/2} = 72^6$.

Pairs → $12 \div 2 =$ **6** ways as a product of two factors. ✅

A number has an **odd** count of factors **only if** it is a perfect square — because one factor (the √) pairs with itself.

Try these

$36 = 2^2 \cdot 3^2 \rightarrow 3\times3 =$ **9**

$100 = 2^2 \cdot 5^2 \rightarrow 3\times3 =$ **9**

$60 = 2^2 \cdot 3 \cdot 5 \rightarrow 3\times2\times2 =$ **12**

$84 = 2^2 \cdot 3 \cdot 7 \rightarrow 3\times2\times2 =$ **12**

Even & Odd factors — $N = 72 = 2^3 \cdot 3^2$

- **Odd** factors = factors of the odd part $3^2 = (2+1) =$ **3**.
- **Even** factors = total − odd = $12 - 3 =$ **9**.
- Ways to write N as a product of two **co-prime** factors = $2^{(\text{distinct primes})-1} = 2^{2-1} =$ **2**

  (1×72, 8×9).

⑧ Division Algorithm

Dividend = Divisor × Quotient + Remainder

with **0 ≤ Remainder < Divisor**. e.g. $47 = 5 \times 9 + 2$

⑨ Remainder Tricks

- Remainder of a **product** = product of the individual remainders (then re-take the remainder).
- **Negative remainder**: if $a \equiv -1 \pmod{d}$, then $a^{even} \equiv +1$ and $a^{odd} \equiv -1$.

Worked example — remainder of $15^{23} \div 16$

Step 1 — $15 = 16 - 1 \therefore 15 \equiv \mathbf{-1} \pmod{16}$.

Step 2 — $15^{23} \equiv (-1)^{23} = -1$ (exponent is odd).

Step 3 — $-1 \pmod{16} = 16 - 1 =$ **15**.

Answer → remainder = **15**. ✅

⑩ Factorial — n!

- Trailing zeros in n! = $\mathbf{[n/5] + [n/25] + [n/125] + \ldots}$
- Highest power of prime **p** in n! = $\mathbf{[n/p] + [n/p^2] + [n/p^3] + \ldots}$

([x] = greatest integer ≤ x, i.e. take the floor / whole part)

Worked example — trailing zeros in 100!

Step 1 — zeros come from factors of $10 = 2 \times 5$; 5's are fewer, so **count 5's**.

Step 2 — $[100/5] + [100/25] + [100/125] = 20 + 4 + 0$.

Answer → $20 + 4 =$ **24 trailing zeros**. ✅

💡 Same method for any prime: highest power of 3 in 100! = 33+11+3+1 = **48**.

⑪ Important Sum Formulas Σ

- Sum of first n naturals: $1+2+\ldots+n$ = **$n(n+1)/2$**
- Sum of squares: $1^2+2^2+\ldots+n^2$ = **$n(n+1)(2n+1)/6$**
- Sum of cubes: $1^3+2^3+\ldots+n^3$ = **$[\,n(n+1)/2\,]^2$**
- Sum of first n **odd** numbers: $1+3+5+\ldots$ = **n^2**
- Sum of first n **even** numbers: $2+4+6+\ldots$ = **$n(n+1)$**
- Count of numbers from a to b = **$b - a + 1$**

✓ Neat link ⟶ sum of cubes = (sum of naturals)^2. So $1^3+2^3+3^3 = (1+2+3)^2 = 6^2 = 36$.

⑫ Quick Facts

✓ A perfect **square never** ends in 2, 3, 7 or 8.

✓ **HCF × LCM = product of the two numbers.**

✓ Squares end only in 0, 1, 4, 5, 6 or 9.

Mini-example — HCF × LCM

Two numbers are 12 and 30. HCF = 6, LCM = 60.

Check → HCF × LCM = 6 × 60 = 360, and 12 × 30 = **360**. ✓ They match.

Exam use ⟶ if the unit digit of a number is 2, 3, 7 or 8, it can **never** be a perfect square — a fast elimination trick.

Plug-in check

$1+2+\ldots+20 = 20\times21/2 =$ **210**

$1^2+\ldots+10^2 = 10\cdot11\cdot21/6 =$ **385**

$1+3+\ldots$(15 odds) $= 15^2 =$ **225**

$2+4+\ldots$(10 evens) $= 10\times11 =$ **110**

Practice Set

1. Unit digit of 3^{65} = ?
2. Unit digit of 2^{54} = ?
3. Unit digit of 7^{123} = ?
4. Is 143 divisible by 11?
5. Is 1001 divisible by 7 & 13?
6. No. of factors of 36 = ?
7. No. of factors of 100 = ?
8. Trailing zeros in 50! = ?
9. Sum 1+2+...+20 = ?
10. Sum of first 15 odd nos. = ?
11. HCF 6, LCM 60, one no. 12; other?
12. How many primes below 100?

Answer Key — with working

1→ 3 (65÷4 rem1 → 1st of 3,9,7,1) · **2→ 4** (54÷4 rem2 → 2nd of 2,4,8,6) · **3→ 3** (123÷4 rem3 → 3rd of 7,9,3,1) · **4→ Yes** ((3+1)−4 = 0; 11×13) · **5→ Yes** (1001 = 7×11×13) · **6→ 9** ($2^2 \cdot 3^2$ → 3x3) · **7→ 9** ($2^2 \cdot 5^2$ → 3x3) · **8→ 12** ([50/5]+[50/25] = 10+2) · **9→ 210** (20×21/2) · **10→ 225** (15^2) · **11→ 30** (6×60/12) · **12→ 25** primes.

Quick Revision

- ✓ N ⊂ W ⊂ Z ⊂ Q ⊂ R; irrationals fill the gaps. 1 is neither prime nor composite; 2 is the only even prime.
- ✓ Unit digit → divide exponent by cycle length; factors → (p+1)(q+1)... ; zeros in n! → count 5's.
- ✓ $\Sigma n = n(n+1)/2$ · $\Sigma n^2 = n(n+1)(2n+1)/6$ · $\Sigma odd = n^2$ · HCF×LCM = product.

✓ Golden rule → before calculating, always **prime-factorise**. Almost every Number-System shortcut starts from $a^p \cdot b^q \cdot c^r$.

☆ Master numbers — the base of all maths! ☆