HCF & LCM

~ Quantitative Aptitude · SSC Exams ~

← circled = key info!

HCF pulls numbers down to the biggest common part; **LCM** stretches them **up to the smallest common multiple**. Every divisibility & grouping question hides one of these two.

① What are HCF & LCM?

HCF (GCD)

Highest Common Factor = the **greatest** number that divides each of the given numbers **exactly** (remainder 0).

HCF(12,18) = 6 — 6 divides both.

LCM

Least Common Multiple = the **smallest** number that is **exactly divisible** by each of the given numbers.

LCM(12,18) = 36 — both divide 36.

- HCF is a **factor** → always ≤ the smallest number.
- LCM is a **multiple** → always ≥ the largest number.
- So for any set: **HCF ≤ smallest ≤ largest ≤ LCM**.

Method 1 — Prime Factorisation

- **HCF** = product of **common** primes taken to their **LOWEST** power.
- **LCM** = product of **all** primes taken to their **HIGHEST** power.

Worked example — HCF & LCM of 72 and 120

Step 1 — factorise: $72 = 2^3 \cdot 3^2$ · $120 = 2^3 \cdot 3 \cdot 5$.

HCF → common primes, lowest powers = $2^3 \cdot 3^1 = $ **24**.

LCM → all primes, highest powers = $2^3 \cdot 3^2 \cdot 5 = $ **360**.

Check → HCF × LCM = 24 × 360 = 8640 = 72 × 120. ✅

✓ Memory hook → HCF = Low (common & lowest), LCM = Large (all & highest). Line the powers up in a column and pick.

② HCF by Euclidean Division ☒

When primes are hard to spot, use **Euclid's method**: divide the larger by the smaller, then divide the previous **divisor** by the **remainder**. Repeat until remainder = 0. The **last divisor** is the HCF.

Rule ⟶ keep dividing divisor ÷ remainder

Last non-zero remainder's divisor = **HCF**. Also called repeated division.

Worked example – HCF of 48 and 60

Step 1 — $60 \div 48 \to 60 = 48 \times 1 + \mathbf{12}$.

Step 2 — now $48 \div 12 \to 48 = 12 \times 4 + \mathbf{0}$.

Step 3 — remainder is 0; last divisor = **12**.

Answer → HCF(48, 60) = **12**. ✅

Bigger numbers – HCF of 1071 and 1547

Step 1 — $1547 = 1071 \times 1 + \mathbf{476}$.

Step 2 — $1071 = 476 \times 2 + \mathbf{119}$.

Step 3 — $476 = 119 \times 4 + \mathbf{0}$ → stop.

Answer → HCF = **119** ($1071 = 119 \times 9$, $1547 = 119 \times 13$). ✅

HCF of **three** numbers ⟶ find HCF of any two first, then take its HCF with the third: HCF(a,b,c) = HCF(HCF(a,b), c).

Chained HCF – quick demo

HCF(24, 36, 60): HCF(24, 36) = 12; then HCF(12, 60) = **12**.

Cross-check by primes: $24 = 2^3 \cdot 3$, $36 = 2^2 \cdot 3^2$, $60 = 2^2 \cdot 3 \cdot 5$ → common $2^2 \cdot 3 = 12$. ✅

✓ Euclid never fails & needs no factorising – ideal when numbers are large or awkward (like 1071 & 1547 above).

③ LCM by Common Division

Write the numbers in a row. Divide by any prime that divides **at least one**; carry unchanged those it doesn't divide. Stop when all become 1. **LCM = product of all the left-side divisors.**

2	12	15	20
2	6	15	10
3	3	15	5
5	1	5	5
✓	**1**	**1**	**1**

LCM of 12, 15, 20

Divisors used → 2, 2, 3, 5.

LCM = 2 × 2 × 3 × 5 = **60**.

By primes: $12 = 2^2{\cdot}3$, $15 = 3{\cdot}5$, $20 = 2^2{\cdot}5$ → highest = $2^2{\cdot}3{\cdot}5 = 60$. ✅

A number that is **not** divisible by the current prime simply **drops down unchanged** (see how 15 stayed 15 through both ÷2 steps).

Second worked — LCM of 8, 9, 12

Primes → $8 = 2^3$, $9 = 3^2$, $12 = 2^2{\cdot}3$.

Highest powers → $2^3 \cdot 3^2 = 8 \times 9 =$ **72**.

Check → 72÷8=9, 72÷9=8, 72÷12=6 — all exact. ✅

Try these — LCM

LCM(4,6) = **12**

LCM(6,8,10) = **120**

LCM(9,12,15) = **180**

LCM(3,4,5,6) = **60**

✓ If one number is a factor of another, the **bigger** one is the LCM: LCM(6, 12) = 12; and the **smaller** is the HCF: HCF(6, 12) = 6.

④ Golden Relations Ʃ

For any TWO numbers a & b

- **HCF × LCM = a × b** (only true for two numbers)
- HCF **always divides** LCM (LCM is a multiple of HCF).
- Co-prime numbers → **HCF = 1** and **LCM = a × b**.
- LCM = (a × b) / HCF · HCF = (a × b) / LCM.

HCF × LCM = a × b works for **exactly two** numbers. It does **NOT** extend to three — never write HCF×LCM = a×b×c.

Product Rule — find the missing number

Worked — HCF = 6, LCM = 60, one number = 12

Step 1 — a × b = HCF × LCM = 6 × 60 = **360**.

Step 2 — other number = 360 ÷ 12.

Answer → other number = **30** [HCF(12,30)=6, LCM(12,30)=60 ✓].

Counting pairs with a given HCF

Worked — product 2028, HCF 13; how many pairs?

Step 1 — let the numbers be 13a, 13b with a, b co-prime.

Step 2 — 13a × 13b = 2028 → ab = 2028 / 169 = **12**.

Step 3 — co-prime pairs with ab = 12: (1,12) & (3,4).

Answer → **2 pairs** — (13, 156) and (39, 52). ✅

Key idea → whenever HCF = h, write the numbers as **h·a** and **h·b** where a, b are **co-prime**. This unlocks almost every HCF word-problem.

⑤ HCF & LCM of Fractions

Two clean formulas

- **HCF of fractions = HCF(numerators) / LCM(denominators)**
- **LCM of fractions = LCM(numerators) / HCF(denominators)**

(fractions must be in lowest terms first)

Worked – HCF & LCM of 3/4, 5/6, 7/8

Numerators 3, 5, 7 → HCF = 1, LCM = 105.

Denominators 4, 6, 8 → HCF = 2, LCM = 24.

HCF of fractions = HCF(num) / LCM(den) = **1/24**.

LCM of fractions = LCM(num) / HCF(den) = **105/2**. ✅

*Note the **cross pattern**: HCF uses **HCF-top & LCM-bottom**; LCM uses **LCM-top & HCF-bottom**. The HCF of fractions is always ≤ every fraction.*

⑥ HCF & LCM of Decimals

Step 1 → give every number the **same** number of decimal places (pad with zeros).

Step 2 → drop the points — treat them as whole numbers.

Step 3 → find HCF / LCM of those integers.

Step 4 → put the decimal point back (same no. of places).

Worked – HCF & LCM of 0.6, 1.2, 0.36

Equalise to 2 places → 0.60, 1.20, 0.36 → integers **60, 120, 36**.

HCF(60,120,36) = 12 → replace point → **0.12**.

LCM(60,120,36) = 360 → replace point → **3.60**. ✅

✓ *Padding matters: 0.6 → 0.60 (i.e. 60), not 6. Keep places equal or the answer's place value goes wrong.*

⑦ Application Formulas – HCF type

- **Greatest** number dividing a, b, c leaving the **SAME** remainder = **HCF of the differences** (a−b), (b−c), (c−a).
- **Greatest** number dividing a, b, c leaving remainders r_1, r_2, r_3 = **HCF of** ($a-r_1$), ($b-r_2$), ($c-r_3$).

Worked – same remainder

Greatest number that divides **43, 91, 183** leaving the **same** remainder.

Differences → 91−43 = 48, 183−91 = 92, 183−43 = 140.

HCF(48, 92, 140) = 4.

Answer → **4** (43, 91, 183 each leave remainder 3). ✅

Worked – different remainders

Greatest number that divides **400** & **435** leaving remainders **9** & **10**.

Subtract remainders → 400−9 = 391, 435−10 = 425.

HCF(391, 425): 391 = 17×23, 425 = 17×25 → HCF = 17.

Answer → **17** (400÷17 rem 9, 435÷17 rem 10). ✅

The word **"greatest number that divides"** almost always means HCF; the word **"least number divisible by"** means LCM. Spot the keyword first!

Real-world HCF – largest equal groups

Largest tape that **exactly** measures 7 m, 3.85 m & 12.95 m → convert to cm: 700, 385, 1295.
HCF(700, 385, 1295) = 35 cm → longest tape = **35 cm**. ✅

✓ "Maximum students in equal rows", "largest tile", "biggest measuring can" – all are HCF problems in disguise.

⑧ Application Formulas — LCM type

- **Least** number leaving the **SAME** remainder r by a, b, c = **LCM(a,b,c) + r**.
- **Least** number leaving remainders r_1, r_2, r_3 where $(a-r_1) = (b-r_2) = (c-r_3) = k$, is **LCM(a,b,c) − k**.
- **Greatest / smallest** n-digit number divisible by x → use the LCM/multiple nearest the boundary.

Same remainder

Least no. leaving rem **3** by 5, 6, 7.
LCM(5,6,7) = 210.
Answer = 210 + 3 = **213**. ✅

Different remainders

Least no. leaving **3,4,5** by 5,6,7.
5−3 = 6−4 = 7−5 = k = 2.
Answer = 210 − 2 = **208**. ✅

Greatest / smallest 4-digit number divisible by 32

Smallest → 1000 ÷ 32 = 31 rem 8; next multiple = 32×32 = **1024**.
Greatest → 9999 ÷ 32 = 312 rem 15; 32×312 = **9984**.

Bells / lights toll together

Bells ring at **6, 9, 12** min. They ring together every **LCM**(6,9,12) = **36 min**.
In 6 hours (360 min) they coincide 360 / 36 = 10 times, plus once at the **start** = **11 times**. ✅

In time T they coincide T/LCM times — and +1 if you count the moment they all start together. Read the question to see if the start is counted.

⑨ Ratio & Product Problems

Two numbers in ratio **a : b** with HCF **h** → numbers are **ah** & **bh**; LCM = **a · b · h**.
Worked — ratio 3 : 4, HCF 4 → numbers 12 & 16; LCM = 3×4×4 = **48**. ✅

Practice Set

1. HCF of 48 and 60 = ?
2. LCM of 12, 15, 20 = ?
3. HCF 6, LCM 60, one no. 12; other?
4. HCF of 3/4, 5/6, 7/8 = ?
5. LCM of 3/4, 5/6, 7/8 = ?
6. Greatest no. dividing 400 & 435 leaving rem 9 & 10?
7. Least no. leaving rem 3 by 5,6,7?
8. Least no. leaving rem 3,4,5 by 5,6,7?
9. Smallest 4-digit no. ÷ by 32?
10. Greatest 4-digit no. ÷ by 32?
11. Bells at 6,9,12 min toll together after?
12. Ratio 3:4, HCF 4; LCM = ?

Answer Key — with working

1→ 12 (60=48×1+12; 48=12×4) · **2→ 60** (2·2·3·5) · **3→ 30** (6×60/12) · **4→ 1/24** (HCF3,5,7=1 / LCM4,6,8=24) · **5→ 105/2** (LCM3,5,7=105 / HCF4,6,8=2) · **6→ 17** (HCF of 391,425) · **7→ 213** (LCM 210 + 3) · **8→ 208** (LCM 210 − 2) · **9→ 1024** (32×32) · **10→ 9984** (32×312) · **11→ 36 min** (LCM 6,9,12) · **12→ 48** (3×4×4).

Quick Revision

- ✓ **HCF = Low** (common primes, lowest powers) · **LCM = Large** (all primes, highest powers) · HCF ≤ smallest ≤ largest ≤ LCM.
- ✓ Two numbers only: **HCF × LCM = a × b** · co-prime → HCF 1, LCM = a×b · write numbers as h·a, h·b (a,b co-prime).
- ✓ Fractions → HCF(num)/LCM(den) & LCM(num)/HCF(den) · "greatest divides" = HCF · "least divisible" = LCM.
- ✓ Same remainder → LCM + r · different (a−r=k) → LCM − k · bells → LCM of intervals, +1 for the start.

✓ Golden rule → read for the keyword. **Groups / largest / exactly divides** → HCF. **Together / least / repeats** → LCM.

☆ HCF & LCM — the pair that unlocks divisibility! ☆