Geometry

~ Quantitative Aptitude · SSC Exams ~

← circled = key info!

Geometry rewards a good figure. Always draw, mark the equal & right angles, then apply a **theorem** — angles, triangles, circles & polygons all follow fixed rules.

① Lines & Angles

Types of angles

Acute → $0° < \angle < 90°$

Right → $\angle = 90°$

Obtuse → $90°–180°$

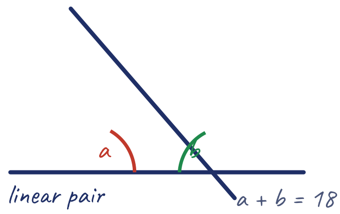
Straight → $\angle = 180°$

Reflex → $180°–360°$

Complete → $360°$

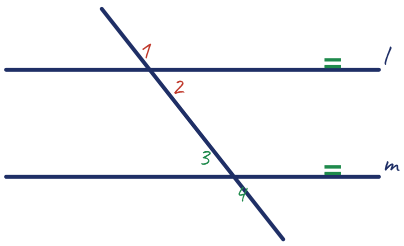
Pairs to know

- **Complementary** → sum = $90°$
- **Supplementary** → sum = $180°$
- **Linear pair** → adjacent, on a line, sum = $180°$
- **Vertically opposite** angles are **equal**.



- Complement of $35°$ = **$55°$**
- Supplement of $130°$ = **$50°$**

Parallel lines cut by a transversal



- **Corresponding** angles → **equal**
- **Alternate** (interior/exterior) → **equal**
- **Co-interior** (same side) → sum **$180°$**

If $\angle 1 = 65°$ → corr. & alt. = **$65°$**, co-interior = **$115°$**.

Worked example — two supplementary angles in ratio 2 : 3

Step 1 — let the angles be $2x$ and $3x$; supplementary ∴ $2x + 3x = 180°$.

Step 2 — $5x = 180° \rightarrow x = 36°$.

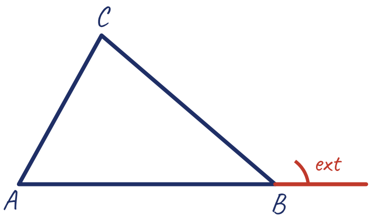
Answer → angles = $2x$ = **$72°$** and $3x$ = **$108°$**. ✅

② Triangles – Angles & Sides △

Angle-sum & exterior-angle theorems

- The three interior angles add to **180°** ($\angle A + \angle B + \angle C = 180°$).
- An **exterior** angle = sum of the two **remote interior** angles.

e.g. interior 50° & 60° → exterior at 3rd vertex = 50 + 60 = **110°**.



By sides: scalene (all diff), isosceles (2 equal), equilateral (all equal, each 60°).

By angles: acute (all <90°), right (one 90°), obtuse (one >90°).

Side rules

- **Triangle inequality** → sum of any two sides > the third side.
- The **side** and the **angle** opposite it grow together → longer side → larger opposite angle.

(3, 4, 5) → 3+4>5 ∴ **valid**

(6, 8, 10) → 6+8>10 ∴ **valid**

(2, 2, 5) → 2+2<5 ∴ **not a triangle**

Sides 5, 7, 9 → largest ∠ opposite **9**

Worked example – angles are x, $x+20°$, $x+40°$

Step 1 — angle-sum: $x + (x+20) + (x+40) = 180°$.

Step 2 — $3x + 60 = 180 \rightarrow 3x = 120 \rightarrow x = 40°$.

Answer → angles = **40°, 60°, 80°** — an acute triangle. ✅

A triangle can have **at most one** right or obtuse angle. In an isosceles triangle the angles opposite the two **equal sides** are equal.

Try these

Ext. angle 120°, one remote 45° → other = **75°**

Can (7, 10, 3) form a △? 7+3=10 → **No**

Right isosceles → angles **90°, 45°, 45°**

∠A=100°, ∠B=40° → ∠C = **40°**

③ Congruence & Similarity

Congruence (≅) – same shape and size

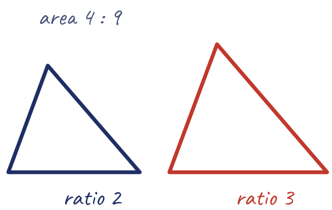
Rule	What must match
SSS	all three pairs of sides equal
SAS	two sides & the **included** angle
ASA	two angles & the **included** side
AAS	two angles & a non-included side
RHS	right angle, hypotenuse & one side

⚠ **AAA** and **SSA** do **not** prove congruence.

Similarity (∼) – same shape, sizes in proportion

Criteria & ratios

- **AA** (two angles equal), **SSS** (sides proportional), **SAS**.
- Ratio of sides = ratio of perimeters = ratio of medians / altitudes.
- **Ratio of areas = (ratio of sides)^2**.



Worked example – $\Delta ABC \sim \Delta DEF$

Given — $AB / DE = 3 / 5$ and area of $\Delta ABC = 54$ cm^2. Find area of ΔDEF.

Step 1 — area ratio $= (3/5)^2 = 9/25 = ar(ABC)/ar(DEF)$.

Step 2 — $ar(DEF) = 54 \times 25 / 9 = 6 \times 25$.

Answer → $ar(DEF) =$ **150 cm^2**. ✅

✓ Two similar triangles with areas 16 & 25 → side ratio = $\sqrt{16} : \sqrt{25}$ = **4 : 5**. A side of 8 in the first → matching side **10** in the second.

Try these

Side ratio 2:3 → area ratio **4:9**

Perimeter ratio 5:7 → side ratio **5:7**

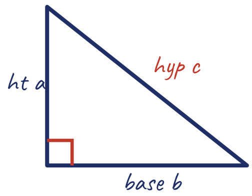
Area ratio 9:16 → side ratio **3:4**

Right angle + hyp + leg → use **RHS**

④ Pythagoras Theorem

Theorem & converse

- In a right triangle: **(hypotenuse)^2 = (base)^2 + (height)^2**.
- **Converse** → if $a^2 + b^2 = c^2$, the angle opposite c is **90°**.



Pythagorean triples — memorise!

3 – 4 – 5	**5 – 12 – 13**
8 – 15 – 17	**7 – 24 – 25**

Any multiple works too — e.g. (3,4,5) × 2 = (6, 8, 10); (5,12,13) × 2 = (10, 24, 26). Also useful: 9–40–41, 20–21–29.

Worked example — ladder against a wall

Given — a 13 m ladder rests with its foot 5 m from the wall. How high does it reach?

Step 1 — height = $\sqrt{(13^2 - 5^2)} = \sqrt{(169 - 25)} = \sqrt{144}$.

Answer → height = **12 m** (it is a 5–12–13 triangle). ✅

Converse check — is (9, 12, 15) right-angled?

$9^2 + 12^2 = 81 + 144 = 225 = 15^2$ ∴ **yes**, right-angled (it is 3–4–5 × 3).

Diagonal of a rectangle = $\sqrt{(l^2 + b^2)}$; diagonal of a square of side a = **$a\sqrt{2}$**; space-diagonal of a cube = **$a\sqrt{3}$**.

Try these

Legs 8 & 15 → hyp = **17**

Legs 6 & 8 → hyp = **10**

Square side 5 → diagonal = **$5\sqrt{2}$**

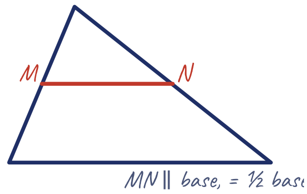
Hyp 25, leg 7 → other leg = **24**

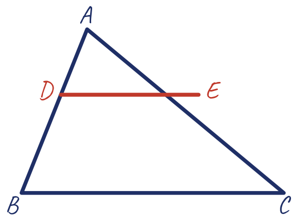
⑤ Mid-point, Thales & Bisector

Mid-point theorem

The segment joining the mid-points of two sides is **parallel** to the third side and **half** its length.

3rd side = 10 → mid-segment = **5**.





Basic Proportionality (Thales)

A line DE ∥ BC divides the other two sides in the same ratio: **AD/DB = AE/EC**.

AD=2, DB=3 → AE:EC = **2:3**.

Angle-bisector theorem

The bisector of ∠A meets BC at D and splits it in the ratio of the adjacent sides: **BD / DC = AB / AC**.

Worked example — angle bisector splits BC

Given — AB = 6, AC = 9, BC = 10; AD bisects ∠A. Find BD and DC.

Step 1 — BD/DC = AB/AC = 6/9 = 2/3 → parts 2 & 3 (total 5).

Step 2 — BD = 10 × 2/5 = 4; DC = 10 × 3/5 = 6.

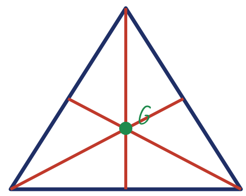
Answer → BD = **4**, DC = **6**. ✅

Mid-point theorem is just Thales with the ratio = 1:1. Joining the mid-points of all three sides makes 4 **congruent** small triangles.

⑥ Centres of a Triangle – I

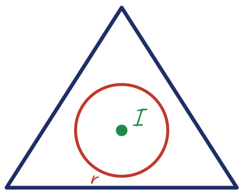
Centroid (G)

- Meeting point of the three **medians** (vertex → mid-point of opposite side).
- Divides each median **2 : 1** (vertex side longer).
- Always lies **inside** the triangle.



Mini-example – centroid on a median

A median is 12 cm long. The centroid divides it 2:1 → the two parts are $12 \times 2/3 =$ **8 cm** (vertex to G) and $12 \times 1/3 =$ **4 cm** (G to base).



Incentre (I)

- Meeting point of the three **angle bisectors**; centre of the **inscribed** circle.
- Equidistant from all three sides → **inradius r = Area / s** (s = semi-perimeter).
- Always lies **inside** the triangle.

Worked example – inradius of a 5–12–13 triangle

Step 1 — it is right-angled ($5^2+12^2=13^2$) → Area $= \frac{1}{2} \times 5 \times 12 = 30$.

Step 2 — $s = (5+12+13)/2 = 15$.

Step 3 — $r = \text{Area}/s = 30/15 = 2$.

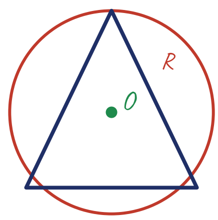
Answer → inradius r = **2 units** (for a right Δ, $r = (a+b-c)/2 = (5+12-13)/2 = 2$ too). ✅

The **centroid** and the **incentre** are always **inside** the triangle – whatever its shape.
Circumcentre & orthocentre can wander outside (next page).

⑥ Centres of a Triangle – II

Circumcentre (O)

- Meeting point of the **perpendicular bisectors** of the sides; centre of the **circumscribed** circle.
- Equidistant from all vertices → **R = abc / (4 × Area)**.



Orthocentre (H)

- Meeting point of the three **altitudes** (perpendiculars from each vertex to the opposite side).

Where it sits	Acute Δ	Right Δ	Obtuse Δ
Centroid / Incentre	inside	inside	inside
Circumcentre	inside	mid-pt of hyp.	outside
Orthocentre	inside	right-angle vertex	outside

Worked example – circumradius of a 6–8–10 triangle

Step 1 — right-angled ($6^2+8^2=10^2$) → Area = ½ × 6 × 8 = 24.

Step 2 — R = abc/(4×Area) = (6 × 8 × 10)/(4 × 24) = 480/96 = 5.

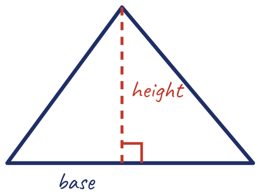
Answer → R = **5 units** (= half the hypotenuse, as expected for a right Δ). ✅

✓ **Euler line** → O, G and H are collinear, and G divides OH in the ratio **1 : 2** (OG : GH). For an equilateral triangle all four centres coincide.

⑦ Area of a Triangle

Key area formulas

- General: Area = ½ × **base** × **height**
- Equilateral (side a): Area = $(\sqrt{3} / 4)\, a^2$, height = $(\sqrt{3} / 2)\, a$
- Heron: Area = $\sqrt{s(s-a)(s-b)(s-c)}$, $s = (a+b+c)/2$
- A **median** splits a triangle into **two equal** areas; the three medians make **6 equal** parts.



Equilateral, $a = 6$

Area = $(\sqrt{3}/4)(6^2) = (\sqrt{3}/4)(36) = 9\sqrt{3}$.

Height = $(\sqrt{3}/2)(6) = 3\sqrt{3}$.

Worked example — Heron for a 13–14–15 triangle

Step 1 — $s = (13+14+15)/2 = 21$.

Step 2 — Area = $\sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056}$.

Answer → Area = **84 sq units**. ✅

Two triangles on the **same base** and between the **same parallels** have **equal area**. Equal bases & equal heights ⟶ equal area, whatever the shape.

Try these

Base 10, ht 6 → area = **30**

Right △ legs 9,12 → area = **54**

Equilateral $a=4$ → area = $4\sqrt{3}$

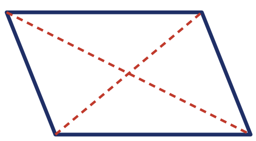
Equilateral $a=10$ → area = $25\sqrt{3}$

⑧ Quadrilaterals

Shape	Sides	Angles	Diagonals
Parallelogram	opp. sides equal & ∥	opp. angles equal	bisect each other
Rectangle	opp. sides equal	all 90°	equal, bisect
Square	all sides equal	all 90°	equal, bisect at 90°
Rhombus	all sides equal	opp. angles equal	unequal, bisect at 90°
Trapezium	one pair ∥	co-int. pair = 180°	unequal
Kite	2 pairs adj. sides equal	one pair equal	cross at 90°, one bisected

Area formulas

- Parallelogram = **base × height** · Rectangle = $\mathbf{l \times b}$ · Square = $\mathbf{a^2 = \frac{1}{2} d^2}$
- Rhombus / Kite = $\mathbf{\frac{1}{2} \times d_1 \times d_2}$ · Trapezium = $\mathbf{\frac{1}{2}(a + b) \times h}$



Rhombus example

Diagonals 6 & 8 → Area = ½ × 6 × 8 = **24**.

Side = $\sqrt{(3^2+4^2)}$ = **5** (half-diagonals meet at 90°).

Worked example — trapezium area

Parallel sides 8 cm & 12 cm, height 5 cm → Area = ½(8+12)(5) = ½ × 20 × 5 = **50 cm²**.

✅

Every square is a rhombus & a rectangle; every rectangle & rhombus is a parallelogram. Sum of all four interior angles = **360°**.

⑨ Polygons ⬠

Formulas for an n-sided polygon

- Sum of interior angles = $(n-2) \times 180°$
- Each interior angle (regular) = $(n-2) \times 180° / n$
- Each exterior angle (regular) = $360° / n$ (all exterior angles sum to $360°$)
- Number of diagonals = $n(n-3) / 2$

Polygon (n)	Interior sum	Each interior	Diagonals
Triangle (3)	180°	60°	0
Quadrilateral (4)	360°	90°	2
Pentagon (5)	540°	108°	5
Hexagon (6)	720°	120°	9
Octagon (8)	1080°	135°	20

Worked example — each interior angle is 150°. Find n.

Step 1 — each exterior = $180° - 150° = 30°$.

Step 2 — $n = 360° / (\text{each exterior}) = 360 / 30 = 12$.

Answer → it is a regular **12-gon** (dodecagon). ✅

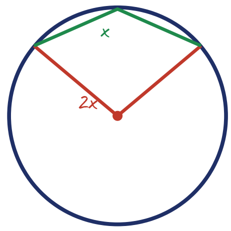
✓ The **exterior** angles of **any** convex polygon always add up to **360°** — no matter how many sides.
A regular polygon with each exterior 24° has $n = 360/24 =$ **15** sides.

⑩ Circles – Angle Theorems

Chord → joins 2 points on circle
Arc → part of the circle
Secant → line cutting at 2 points
Segment → chord + arc region
Tangent → touches at 1 point
Sector → two radii + arc

Four key theorems

- Angle at **centre = 2 × angle** at circumference on the same arc.
- Angles in the **same segment** are equal.
- Angle in a **semicircle = 90°**.
- **Cyclic quad.** → opposite angles sum to **180°**.



Worked example – central vs inscribed angle

Given — an arc subtends 100° at the centre. What angle does it make at the circumference?

Rule — inscribed angle = ½ × central angle = ½ × 100°.

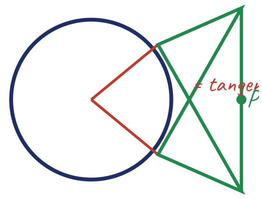
Answer → **50°**. ✅

Worked example – cyclic quadrilateral ABCD

$\angle A = 70°$ and $\angle B = 80°$. Opposite angles sum to 180° ∴ $\angle C = 180-70 =$ **110°** and $\angle D = 180-80 =$ **100°**. ✅

Equal chords are equidistant from the centre; the perpendicular from the centre **bisects** the chord. The angle in a semicircle is a right angle – the diameter is the longest chord.

⑪ Tangents & Chords



Core results

- A tangent is ⊥ to the radius at the point of contact.
- The **two tangents** drawn from an external point are **equal** in length.
- **Alternate segment** → angle between tangent & chord = angle in the alternate segment.

Power-of-a-point formulas

- Two chords crossing inside at P: **$PA \times PB = PC \times PD$**.
- Tangent & secant from external P: **$PT^2 = PA \times PB$**.
- Tangent length from external P: **$PT = \sqrt{(d^2 - r^2)}$** (d = distance to centre).

Worked example — tangent-secant

Given — from P a tangent PT and a secant hit the circle at A (near) & B (far), with PA = 4, PB = 9. Find PT.

Step 1 — $PT^2 = PA \times PB = 4 \times 9 = 36$.

Answer → $PT = \sqrt{36} =$ **6**. ✅

Worked example — intersecting chords

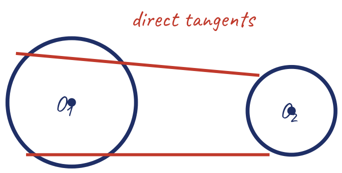
Chords AB & CD meet at P inside the circle. PA = 3, PB = 8, PC = 4 → PD = (PA × PB) / PC = 24 / 4 = **6**. ✅

💡 Tangent from a point 13 units from the centre of a circle of radius 5: $PT = \sqrt{(13^2 - 5^2)} = \sqrt{144} =$ **12** (a 5–12–13 triangle again!).

⑫ Two Circles — Common Tangents

Tangent-length formulas (d = distance between centres)

- **Direct** common tangent (DCT): length = $\sqrt{d^2 - (r_1 - r_2)^2}$
- **Transverse** common tangent (TCT): length = $\sqrt{d^2 - (r_1 + r_2)^2}$



No. of common tangents

- Separate ($d > r_1+r_2$) → **4**
- Touch externally ($d = r_1+r_2$) → **3**
- Intersecting → **2**
- Touch internally ($d = |r_1-r_2|$) → **1**
- One inside other → **0**

Worked example — direct common tangent

Given — radii $r_1 = 8$, $r_2 = 3$; centres 13 apart. Find the DCT length.

Step 1 — DCT = $\sqrt{13^2 - (8 - 3)^2} = \sqrt{169 - 25} = \sqrt{144}$.

Answer → DCT = **12 units**. ✅

Worked example — transverse common tangent

Given — radii $r_1 = 4$, $r_2 = 2$; centres 10 apart. Find the TCT length.

Step 1 — TCT = $\sqrt{10^2 - (4 + 2)^2} = \sqrt{100 - 36} = \sqrt{64}$.

Answer → TCT = **8 units**. ✅

✓ Transverse tangents exist **only** when the circles are fully apart ($d > r_1+r_2$). The DCT always ≥ TCT because we subtract the **smaller** quantity (r_1-r_2).

Geometry Facts Recap

Triangle centres at a glance

Centre	Made from	Key fact
Centroid G	medians	divides each median 2 : 1
Incentre I	angle bisectors	r = Area / s
Circumcentre O	perp. bisectors	$R = abc$ / (4·Area)
Orthocentre H	altitudes	O, G, H collinear (Euler)

Polygon & angle formulas

Quantity	Formula
Sum of interior angles	$(n - 2) \times 180°$
Each interior (regular)	$(n - 2) \times 180° / n$
Each exterior (regular)	$360° / n$
Number of diagonals	$n(n - 3) / 2$

Circle theorems & areas

Result	Value / formula
Central vs inscribed angle	**central = 2 × inscribed**
Angle in a semicircle	$90°$
Cyclic quad. opposite angles	**sum = 180°**
Tangent & secant from P	$PT^2 = PA \times PB$
Circle area / circumference	πr^2 / $2\pi r$

Memorise: equilateral area $(\sqrt{3}/4)a^2$, rhombus/kite $\frac{1}{2}d_1d_2$, trapezium $\frac{1}{2}(a+b)h$ — these appear in almost every SSC paper.

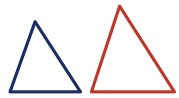
Solved Examples

1. Exterior angle of a triangle

An exterior angle is $110°$ and one remote interior is $45°$. The other remote interior $= 110 - 45 =$ **65°**; the adjacent interior $= 180 - 110 =$ **70°**.

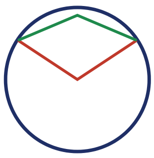
2. Similar triangles

$\Delta PQR \sim \Delta XYZ$ with $PQ:XY = 2:5$. If area of $\Delta PQR = 12$, then area of $\Delta XYZ = 12 \times (5/2)^2 = 12 \times 25/4 =$ **75**.



3. Pythagoras application

A rectangle is 24 cm by 7 cm. Its diagonal $= \sqrt{(24^2 + 7^2)} = \sqrt{(576 + 49)} = \sqrt{625} =$ **25 cm** (a 7–24–25 triple).



4. Circle – inscribed angle

A chord subtends $130°$ at the centre. The angle it makes at any point on the **major** arc $= ½ \times 130° =$ **65°**.

5. Regular polygon

Interior angle of a regular polygon is $140° \rightarrow$ exterior $= 40° \rightarrow n = 360/40 =$ **9** sides (nonagon). Diagonals $= 9(9-3)/2 =$ **27**.

6. Tangent length

Two circles of radii 9 and 4 have centres 13 apart. Direct common tangent $= \sqrt{(13^2 - (9-4)^2)} = \sqrt{(169 - 25)} = \sqrt{144} =$ **12**.

✓ *Habit for exams → sketch, label the known angles/sides, spot the triple or theorem, then compute. A neat figure solves half the problem.*

Practice Set

1. Supplement of 118° = ?
2. Angles of a Δ are x, 2x, 3x. Find them.
3. Legs 20 & 21 → hypotenuse = ?
4. ΔABC ~ ΔDEF, sides 3:4, ar(ABC)=36; ar(DEF)?
5. Interior angle sum of an octagon?
6. Each exterior angle of a regular decagon?
7. Diagonals of a hexagon = ?
8. Central angle 84° → inscribed angle?
9. Cyclic quad: ∠A = 95°; find ∠C.
10. AB=6, AC=9 bisector on BC=10; BD, DC?
11. Equilateral side 8 → area = ?
12. Rhombus diagonals 10 & 24 → side = ?
13. Median 15 → centroid parts?
14. DCT: r=6,2, d=? for length 8. (find d)

Answer Key — with working

1→ 62° (180−118) · **2→ 30,60,90** (6x=180) · **3→ 29** (20-21-29 triple) · **4→ 64** ($36\times(4/3)^2$) · **5→ 1080°** ((8−2)×180) · **6→ 36°** (360/10) · **7→ 9** (6×3/2) · **8→ 42°** (½×84) · **9→ 85°** (180−95) · **10→ BD=4, DC=6** (6:9 = 2:3 of 10) · **11→ 16√3** ((√3/4)·64) · **12→ 13** ($\sqrt{5^2+12^2}$) · **13→ 10 & 5** (2:1 of 15) · **14→ d=√80=4√5** ($8^2=d^2-4^2$).

Quick Revision

- ✓ Δ angle-sum 180°; exterior = sum of remote interiors; ratio of areas of similar Δs = $(\text{side ratio})^2$.
- ✓ Centres: centroid 2:1, r = Area/s, R = abc/4·Area; polygon interior sum (n−2)180°, diagonals n(n−3)/2.
- ✓ Circle: central = 2×inscribed, semicircle 90°, cyclic opp. 180°, $PT^2 = PA\times PB$; DCT = $\sqrt{d^2-(r_1-r_2)^2}$.

☆ *Lines, triangles, circles — geometry mastered!* ☆