Coordinate Geometry

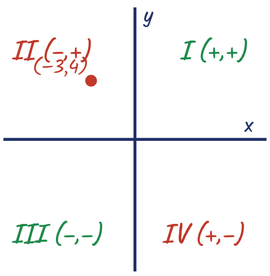
~ Quantitative Aptitude · SSC Exams ~

← circled = key formulas!

Coordinate Geometry describes points, lines & shapes using numbers (x,y). Master **distance, section, slope & line-equation formulas** — almost every SSC question is a direct plug-in.

① The Cartesian Plane & Quadrants

- **Coordinate axes:** the horizontal **x-axis** & vertical **y-axis**, meeting at right angles at the **origin O(0,0)**.
- **Abscissa** = x-coordinate (⊥ distance from y-axis). **Ordinate** = y-coordinate (⊥ distance from x-axis).
- A point is written as an ordered pair **(x, y)**. y = a (constant) is a line ∥ **x-axis**; x = a is a line ∥ **y-axis**.



Sign rule (memorise!)

Q I: **(+,+)** Q II: **(−,+)**

Q III: **(−,−)** Q IV: **(+,−)**

Go anti-clockwise from Q I; only one sign flips each step.

Worked example — locate the quadrants

Step 1 — Point A(−3, 5): x<0, y>0 → matches (−,+).

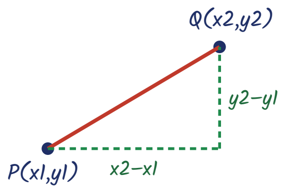
Step 2 — Point B(4, −2): x>0, y<0 → matches (+,−).

Answer → A lies in **Quadrant II**, B lies in **Quadrant IV**. ✅

Points on the axes belong to **no quadrant**: (a,0) is on the x-axis, (0,b) is on the y-axis, and (0,0) is the origin.

② The Distance Formula

Distance between two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is the hypotenuse of a right triangle formed by Δx and Δy — a direct Pythagoras application.



Distance Formula

$$PQ = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$$

Distance from origin to (x,y): $\sqrt{(x^2+y^2)}$

Worked example — distance between (2,3) and (7,15)

Step 1 — $\Delta x = 7-2 = 5$, $\Delta y = 15-3 = 12$.

Step 2 — $PQ = \sqrt{(5^2+12^2)} = \sqrt{(25+144)} = \sqrt{169}$.

Answer → PQ = **13 units** (the 5-12-13 triple!). ✅

Distance from the origin

- Distance of (6,8) from O(0,0) = $\sqrt{(6^2+8^2)} = \sqrt{100} =$ **10**.
- Distance of (−5,12) from O = $\sqrt{(25+144)} = \sqrt{169} =$ **13**.

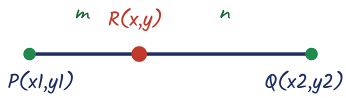
Try these

- Distance between (1,1) and (4,5): $\sqrt{(3^2+4^2)} =$ **5**.
- Distance between (−2,3) and (2,0): $\sqrt{(4^2+3^2)} =$ **5**.

Watch for the Pythagorean triples 3-4-5, 5-12-13, 8-15-17 — SSC loves setting Δx, Δy as one of these pairs to save you the square root!

③ Midpoint & Section Formula

The **section formula** finds the point that divides segment PQ in a given ratio **m : n**. The **midpoint** is just the special case $m = n = 1$.



Section Formula (internal, m:n)

$R(x,y) = \left((mx_2+nx_1)/(m+n) \, , \, (my_2+ny_1)/(m+n) \right)$

Midpoint Formula (m=n=1:1)

Midpoint $= \left((x_1+x_2)/2 \, , \, (y_1+y_2)/2 \right)$

Worked example — midpoint of (2,3) and (8,11)

Step 1 — $x = (2+8)/2 = 5$.

Step 2 — $y = (3+11)/2 = 7$.

Answer → Midpoint = **(5, 7)**. ✅

Worked example — divide (1,2) & (7,11) internally in 2:1

Step 1 — $x = (2\times7+1\times1)/(2+1) = 15/3 = 5$.

Step 2 — $y = (2\times11+1\times2)/3 = 24/3 = 8$.

Answer → R = **(5, 8)**. ✅

Ratio letter order matters: m goes with the **far** point $Q(x_2,y_2)$, n goes with the **near** point $P(x_1,y_1)$ — "m near Q, n near P".

④ External Division & Centroid

External Division (ratio m:n)

$R(x,y) = \left((mx_2 - nx_1)/(m-n) \,,\, (my_2 - ny_1)/(m-n) \right)$

Same as internal, just swap + for − — R now lies **outside** segment PQ.

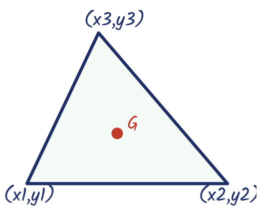
Worked example – divide (1,2) & (7,11) externally in 2:1

Step 1 — $x = (2\times7 - 1\times1)/(2-1) = 13/1 = 13$.

Step 2 — $y = (2\times11 - 1\times2)/1 = 20$.

Answer → R = **(13, 20)**. ✅

Centroid of a Triangle



Centroid Formula

$G = \left((x_1+x_2+x_3)/3 \,,\, (y_1+y_2+y_3)/3 \right)$

Average of all three vertices — the "balance point".

Worked example – centroid of (2,3),(4,7),(6,2)

Step 1 — $x = (2+4+6)/3 = 12/3 = 4$.

Step 2 — $y = (3+7+2)/3 = 12/3 = 4$.

Answer → G = **(4, 4)**. ✅

💡 If the centroid & two vertices are given, the third vertex is found by simple reverse substitution – a very common SSC question type.

⑤ Area of a Triangle & Collinearity

Area Formula

Area = ½ $|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$

*Always take the **modulus** — area can never be negative.*

Worked example — area of (1,1), (4,1), (4,5)

Step 1 — Area = ½|1(1−5)+4(5−1)+4(1−1)|.

Step 2 — = ½|−4+16+0| = ½×12.

Answer → Area = **6 sq units** (check: right Δ, legs 3&4 → ½×3×4=6 ✓). ✅

Collinearity Test

Three points are **collinear** (lie on one straight line) ⇔ the "triangle" they form has **Area = 0**.

Worked example — are (1,2),(3,6),(5,10) collinear?

Area = ½|1(6−10)+3(10−2)+5(2−6)| = ½|−4+24−20| = ½×0 = **0** → **collinear** (all lie on $y=2x$). ✅

Try these

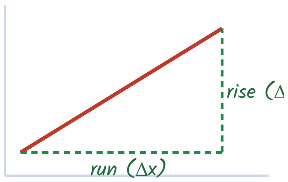
Area of (0,0),(5,0),(0,4) = **10**

Area of (2,1),(4,1),(4,4) = **4.5**

💡 *To find an unknown coordinate that makes 3 points collinear, simply set the **Area formula = 0** and solve — a one-line linear equation.*

⑥ Slope of a Line

*Slope (gradient) measures how **steep** a line is — the "rise over run" between any two points on it.*



Slope Formula

$m = (y_2 - y_1) \;/\; (x_2 - x_1)$

$= \tan\theta$, θ = angle the line makes with $+x$-axis.

Worked example — slope through (2,3) and (6,11)

Step 1 — $m = (11-3)/(6-2) = 8/4$.

Answer → $m =$ **2** (steep, rising left-to-right). ✅

Four types of slope

Line direction	Slope	Example
Rising (↖↗)	**Positive**	(1,1)&(3,5) → m=2
Falling (↙↘)	**Negative**	(1,5)&(3,1) → m=−2
Horizontal (↔)	**0**	(3,4)&(7,4) → m=0
Vertical (↑↓)	**Undefined**	(1,5)&(1,9) → ½/0

💡 *Slope of line $ax+by+c=0$ is simply **$-a/b$** — no need to rearrange to $y=mx+c$ every time.*

⑦ Equations of a Line

Four standard forms

- **Slope-intercept:** $y = mx + c$ (c = y-intercept)
- **Point-slope:** $y - y_1 = m(x - x_1)$
- **Two-point:** $y-y_1 = [(y_2-y_1)/(x_2-x_1)](x-x_1)$
- **Intercept form:** $x/a + y/b = 1$ (a = x-intercept, b = y-intercept)

Worked — line through (2,3), slope 4

Step 1 — $y-3 = 4(x-2)$.

Step 2 — $y-3 = 4x-8 \rightarrow y = 4x-5$.

Answer → $y = 4x - 5$. Check at $x=2$: $y=8-5=3$ ✓ ✅

Worked — write $3x+4y=12$ in intercept form

Step 1 — divide throughout by 12: $x/4 + y/3 = 1$.

Answer → x-intercept = **4**, y-intercept = **3**. ✅

Two-point form — example

Line through (1,2) & (3,8): $m = (8-2)/(3-1) = 3$. Using (1,2): $y-2=3(x-1) \rightarrow y = 3x - 1$.

💡 For line $ax+by+c=0$: x-intercept = $-c/a$, y-intercept = $-c/b$ — set the other variable to 0 mentally.

⑧ Parallel, Perpendicular & Angle

The two golden conditions

- **Parallel** lines ⇒ $m_1 = m_2$ (same slope, never meet).
- **Perpendicular** lines ⇒ $m_1 \times m_2 = -1$ (slopes are negative reciprocals).

Angle between two lines

$\tan\theta = \left| (m_1 - m_2) / (1 + m_1 m_2) \right|$

Worked — are lines with $m=2$ and $m=-½$ perpendicular?

Step — $m_1 \times m_2 = 2 \times (-½) = -1$.

Answer → Product $= -1$ ∴ **Yes, perpendicular.** ✅

Worked — angle between lines $m_1=1$, $m_2=0$

$\tan\theta = |(1-0)/(1+0)| = 1 \rightarrow \theta = 45°$ (a line of slope 1 meets the x-axis at 45°). ✅

Try these

Line ∥ to $y=3x+2$ has slope **3**

Line ⊥ to $y=3x+2$ has slope **$-1/3$**

$2x+3y=6$ & $4x+6y=5$ are **parallel**

$x-y=1$ & $x+y=5$ are **perpendicular**

💡 Two lines $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$ are parallel if $a_1/a_2 = b_1/b_2$ — no slope needed!

⑨ Distance from a Line

Point-to-line distance

$D = |\, ax_1+by_1+c \,| \,/\, \sqrt{(a^2+b^2)}$ for point (x_1,y_1), line $ax+by+c=0$

Distance between two parallel lines

$D = |\, c_1 - c_2 \,| \,/\, \sqrt{(a^2+b^2)}$ for $ax+by+c_1=0$ & $ax+by+c_2=0$

Worked — distance of (2,3) from 3x+4y−6=0

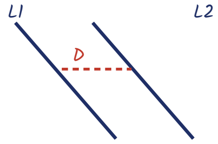
Step 1 — numerator = $|3(2)+4(3)-6| = |6+12-6| = 12$.

Step 2 — denominator = $\sqrt{(3^2+4^2)} = 5$.

Answer → $D = 12/5 =$ **2.4 units**. ✅

Worked — distance between 3x+4y+2=0 & 3x+4y−8=0

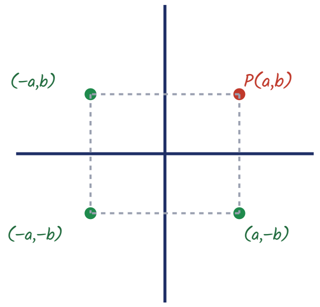
$D = |2-(-8)| \,/\, \sqrt{(9+16)} = 10/5 =$ **2 units**. ✅



Both parallel lines must have the same **a, b** (rewrite one if needed) before using the shortcut $D = |c_1-c_2|/\sqrt{(a^2+b^2)}$.

⑩ Reflection of a Point

A reflection is a **mirror image** across an axis or line — the mirror-line becomes the perpendicular bisector of the segment joining a point and its image.



Reflection Rules for P(a,b)

In x-axis → **(a, −b)**

In y-axis → **(−a, b)**

In origin → **(−a, −b)**

In line y=x → **(b, a)**

Worked — reflect (3, −5)

x-axis → **(3,5)** · y-axis → **(−3,−5)** · origin → **(−3,5)** · y=x → **(−5,3)**. ✅

Try these

(4,7) in x-axis → **(4,−7)**

(−2,6) in y-axis → **(2,6)**

(1,−1) in origin → **(−1,1)**

(5,9) in y=x → **(9,5)**

💡 Reflecting twice through the origin (or x-axis then y-axis) brings you back to **the same sign pattern as a 180° rotation** — (a,b)⟶(−a,−b).

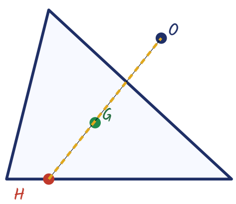
⑪ Special Points of a Triangle

- **Centroid (G)** — meeting point of medians; average of vertices.
- **Circumcenter (O)** — equidistant from all 3 vertices; meeting of perpendicular bisectors of sides.
- **Orthocenter (H)** — meeting point of the three altitudes.
- **Incentre (I)** — equidistant from all 3 sides; meeting of angle bisectors.

Incentre Formula

$$I = \left((ax_1+bx_2+cx_3)/(a+b+c) \,,\, (ay_1+by_2+cy_3)/(a+b+c) \right)$$

a,b,c = side lengths **opposite** to vertices A,B,C.



Euler Line

H, G, O are **collinear**. G divides HO in ratio **2:1**:

$$G = (2O + H) / 3$$

Worked — H=(3,3), O=(0,0), find G

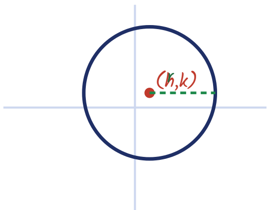
$G = (2(0,0)+(3,3))/3 = (3,3)/3 =$ **(1,1)**. ✅

Worked — incentre of right Δ A(0,0),B(4,0),C(0,3)

Sides: a(opp A)=BC=5, b(opp B)=AC=3, c(opp C)=AB=4. $I = ((5\cdot0+3\cdot4+4\cdot0)/12,$ $(5\cdot0+3\cdot0+4\cdot3)/12) =$ **(1, 1)**. ✅

⑫ Circle – Basic Equation

A circle is the set of points at a fixed **radius r** from a fixed **centre (h,k)**. SSC mostly asks you to read off centre & radius.



Standard form

$(x-h)^2 + (y-k)^2 = r^2$

General form

$x^2+y^2+2gx+2fy+c=0$

Centre = $(-g,-f)$, radius = $\sqrt{(g^2+f^2-c)}$

Worked – centre & radius of $x^2+y^2-6x+8y-11=0$

Step 1 — $2g=-6 \Rightarrow g=-3$; $2f=8 \Rightarrow f=4$; $c=-11$.

Step 2 — centre = $(-g,-f)$ = $(3,-4)$; radius = $\sqrt{(9+16+11)} = \sqrt{36}$.

Answer → centre **(3,−4)**, radius **6**. ✅

Try these

- Circle with centre (2,−3), radius 5: $(x-2)^2+(y+3)^2=25$.
- $x^2+y^2=49$ has centre **(0,0)**, radius **7**.

In general form, the coefficients of x^2 and y^2 must both be **1** (equal & no xy term) before reading off g, f, c – divide through first if not.

Solved Examples

Q1. Distance between (3,4) and (−3,−4).
$\sqrt{[(3-(-3))^2+(4-(-4))^2]} = \sqrt{(36+64)} = \sqrt{100} =$ **10**. ✅

Q2. Point dividing (2,−2) & (−7,4) internally in ratio 1:2.
$x=(1\times-7+2\times2)/3 = -3/3 = -1$; $y=(1\times4+2\times-2)/3 = 0/3 = 0 \rightarrow$ **(−1, 0)**. ✅

Q3. Slope of the line perpendicular to 2x−3y+6=0.
Slope of given line $= -a/b = 2/3$. Perpendicular slope $=$ **−3/2**. ✅

Q4. Find k so that (0,1), (2,5), (k,11) are collinear.
$0(5-11)+2(11-1)+k(1-5)=0 \rightarrow 20-4k=0 \rightarrow$ **k = 5**. ✅

Q5. Equation of the line through (1,2) and (3,8).
$m=(8-2)/(3-1)=3$; $y-2=3(x-1) \rightarrow$ **y = 3x − 1**. ✅

Q6. Centroid of (2,1),(x,3),(4,y) is (3,2). Find x+y.
$(6+x)/3=3 \Rightarrow x=3$; $(4+y)/3=2 \Rightarrow y=2$. $x+y =$ **5**. ✅

✓ Golden move (Q4 & Q6) ⟶ when one coordinate is unknown, plug the given formula and solve the resulting **one-variable linear equation** — almost every SSC question ends this way.

⑬ High-Frequency SSC Shortcuts

Instant reads from a line's equation ax+by+c=0

- Slope = $-a/b$ · x-intercept = $-c/a$ · y-intercept = $-c/b$
- Parallel test (no slope needed): $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Worked — read off line 3x+4y−12=0

Slope = −3/4; x-intercept = 12/3 = **4**; y-intercept = 12/4 = **3**. ✅

Parallelogram test

ABCD is a parallelogram ⇔ **midpoint of diagonal AC = midpoint of diagonal BD**.

Worked — A(1,2),B(4,3),C(6,7),D(3,6): is ABCD a ■ gram?

Mid AC = (3.5, 4.5); Mid BD = (3.5, 4.5). Equal → **Yes, parallelogram**. ✅

Right-angle-triangle incentre shortcut

If the right angle is at the origin with legs along the axes, the **incentre sits at (r, r)**, where r = inradius = (sum of legs − hypotenuse)/2.

💡 *To check if 3 points make a **right triangle**, find all 3 slopes — if any two multiply to −1, that's the right angle. No need to compute all 3 side lengths.*

Practice Set

1. Distance between (0,0) & (6,8)? (a) 10 (b) 14 (c) 8 (d) 6

2. Midpoint of (2,3) & (8,11)? (a) (5,7) (b) (4,6) (c) (6,8) (d) (5,8)

3. Centroid of (1,2),(3,4),(5,6)? (a) (3,4) (b) (2,3) (c) (4,5) (d) (3,3)

4. Slope through (2,3) & (5,9)? (a) 1 (b) 2 (c) 3 (d) ½

5. Which point lies in Quadrant III? (a) (2,−3) (b) (−2,3) (c) (−2,−3) (d) (2,3)

6. Area of Δ(0,0),(4,0),(0,3)? (a) 6 (b) 12 (c) 7 (d) 3.5

7. Line ⊥ to $y=2x+3$ has slope? (a) 2 (b) −2 (c) ½ (d) −½

8. x-intercept of $4x+5y=20$? (a) 5 (b) 4 (c) 20 (d) 1

9. Reflection of (4,−7) in the y-axis? (a) (−4,−7) (b) (4,7) (c) (−4,7) (d) (7,−4)

10. Distance of (3,4) from line $3x+4y-10=0$? (a) 1 (b) 2 (c) 3 (d) 0.6

11. Points (1,1),(2,2),(3,3) are: (a) Collinear (b) A triangle (c) Concyclic (d) None

12. Midpoint of (1,1) & (5,5)? (a) (2,2) (b) (3,3) (c) (4,4) (d) (1,1)

13. Lines with $m_1=3$, $m_2=3$ are: (a) Perpendicular (b) Parallel (c) Meet at 90° (d) None

14. Line of slope 0 through (2,5)? (a) $x=2$ (b) $y=5$ (c) $x=5$ (d) $y=2$

Answer Key — with working

1→(a) √(36+64)=10 · **2→(a)** ((2+8)/2,(3+11)/2) · **3→(a)** avg of vertices · **4→(b)** (9−3)/(5−2) · **5→(c)** (−,−) · **6→(a)** ½×4×3 · **7→(d)** neg. reciprocal · **8→(a)** y=0⇒x=5 · **9→(a)** (−x,y) · **10→(c)** |9+16−10|/5 · **11→(a)** area=0 · **12→(b)** midpoint · **13→(b)** equal slopes · **14→(b)** horizontal line.

Exam file — recent SSC CGL flavour

- Find the x-intercept of $5x+2y=20$. → $y=0 \Rightarrow 5x=20 \Rightarrow x =$ **4**.
- In Δ ABC, B(6,2), C(3,7), centroid G(4,4). Find A. → $(x+6+3)/3=4 \Rightarrow x=3$; $(y+2+7)/3=4 \Rightarrow y=3 \Rightarrow$ A = **(3,3)**.
- Slope of line ⊥ to $y=-5x+2$. → $m_1=-5 \Rightarrow m_2 =$ **1/5**.

Extra Solved Examples

A. Find the ratio in which (2,y) divides (1,3) & (4,9); also find y.

Let ratio = k:1. x: $2=(4k+1)/(k+1) \Rightarrow 2k+2=4k+1 \Rightarrow k=½ \Rightarrow$ ratio **1:2**. Then $y=(k\cdot 9+3)/(k+1)=(4.5+3)/1.5=$ **5**. ✅

B. Distance between parallel lines 5x+12y−4=0 & 5x+12y+9=0.

$\sqrt{(25+144)}=13$. $D = |-4-9|/13 = 13/13 =$ **1 unit**. ✅

C. P(x,3) is equidistant from A(2,1) & B(6,5). Find x.

$(x-2)^2+4 = (x-6)^2+4 \Rightarrow -4x+4=-12x+36 \Rightarrow 8x=32 \Rightarrow$ **$x = 4$**. ✅

Quick Revision

- ✓ Quadrants: I(+,+) II(−,+) III(−,−) IV(+,−). Distance = $\sqrt{(\Delta x^2+\Delta y^2)}$. Midpoint avgs coords; Section = $(mx_2 \pm nx_1)/(m \pm n)$.
- ✓ Centroid = avg of 3 vertices. Area = $½|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$; collinear ⇔ Area=0.
- ✓ Slope $m=\Delta y/\Delta x = -a/b$. Parallel: $m_1=m_2$. Perpendicular: $m_1m_2=-1$. Point-line $D=|ax_1+by_1+c|/\sqrt{(a^2+b^2)}$.
- ✓ Reflections: x-axis(a,−b), y-axis(−a,b), origin(−a,−b), y=x(b,a). Euler line: G=(2O+H)/3.

☆ Coordinate Geometry — plot it, don't fear it! ☆

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