Boat & Stream

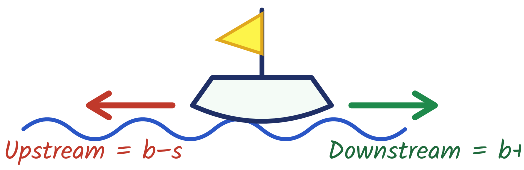
~ Quantitative Aptitude · SSC Exams ~

← circled = key formulas!

Boat & Stream is just Time-Speed-Distance with a twist: the current speeds you up going downstream & slows you down going upstream. Master four formulas — almost every SSC question falls to them.

① Basic Definitions & Key Formulas

Let **b** = speed of the boat (or man) in **still water**, and **s** = speed of the **stream** (current). Going **with** the current is downstream; going **against** it is upstream.



The two master formulas

Downstream speed $D = b + s$

Upstream speed $U = b - s$

Current **helps** downstream, **opposes** upstream.

Worked example — boat 8 km/h, stream 2 km/h

Step 1 — $D = b+s = 8+2 = 10$ km/h; $U = b-s = 8-2 = 6$ km/h.

Step 2 — time to cover 30 km downstream = distance / speed = 30 / 10.

Answer → time = **3 hours**. ✅

Always $s < b$ in real problems — if the stream were faster than the boat, it could never row upstream at all!

② Finding b & s from D & U ← →

Turn the two formulas around: if a question **gives** you the downstream speed D and upstream speed U, you can recover b and s directly — no simultaneous equations needed.

The reverse formulas

- Still-water speed **$b = (D + U) / 2$** (b is the AVERAGE of D & U)
- Stream speed **$s = (D - U) / 2$** (s is HALF the DIFFERENCE)

Worked example — a boat covers 24 km downstream in 2 h and 24 km upstream in 3 h

Step 1 — $D = 24/2 = 12$ km/h; $U = 24/3 = 8$ km/h.

Step 2 — $b = (12+8)/2 = 10$ km/h.

Answer → b = **10 km/h**, $s = (12-8)/2$ = **2 km/h**. ✅

Mini-example — $D = 14$ km/h, $U = 8$ km/h given directly

$b = (14+8)/2$ = **11 km/h**; $s = (14-8)/2$ = **3 km/h**. Check: $11+3=14$ ✓, $11-3=8$ ✓. ✅

Try these

D=16, U=10 → b=**13**, s=**3**

D=22, U=14 → b=**18**, s=**4**

D=9, U=5 → b=**7**, s=**2**

D=30, U=18 → b=**24**, s=**6**

✓ Sanity check every answer: b must always be **bigger** than s. If your s comes out ≥ b, re-check your subtraction!

If the question gives distance & time instead of speed directly, first find D = dist/time(down) and U = dist/time(up) — then apply the b, s formulas.

③ Time = Distance ÷ Speed

Every boat question reduces to the oldest formula in the book: **time = distance / speed**. Just remember to use D for the downstream leg and U for the upstream leg.

Core relations

- t_{down} = distance / D · t_{up} = distance / U
- Total time for a round trip = $t_{down} + t_{up}$

Worked example — b=15 km/h, s=3 km/h, distance 40 km each way

Step 1 — D = 15+3 = 18 km/h; U = 15−3 = 12 km/h.

Step 2 — t_{down} = 40/18 = 5/3 h; t_{up} = 40/12 = 5/2 h.

Answer → total = 5/3 + 5/2 = 10/6+15/6 = 25/6 h = **4 h 10 min**. ✅

Mini-example — b=6 km/h, s=2 km/h, one-way distance 16 km, round trip time?

D=8, U=4 → t_{down}=16/8=2h, t_{up}=16/4=4h ∴ total = **6 hours**. ✅

Try these

b=10,s=2, d=24 → t_{down}=**2h**

b=10,s=2, d=24 → t_{up}=**3h**

b=7,s=1, d=18 → total=**4.5h**

b=13,s=3, d=32 → total=**4h**

To turn a fraction of an hour into minutes, **multiply the fraction by 60**. E.g. 1/3 h × 60 = 20 min, so 4/3 h = 4 h 20 min.

Upstream always takes **longer** than downstream for the same distance, since U < D. A negative or zero t_{up} means the data is wrong.

④ Average Speed for a Round Trip

When the **same distance** is covered downstream and upstream, average speed is **total distance / total time** — the harmonic mean of D and U, **not** their simple average.

The formula

Average speed = **2DU / (D+U)** = **$b - s^2/b$**

Common trap: it is **NOT** (D+U)/2. That formula only works for a return trip at equal speeds each way — here the speeds differ.

Worked example — D=15 km/h, U=10 km/h

Step 1 — avg = 2×15×10 / (15+10) = 300/25.

Step 2 — Check: for d=30 km each way, t_{down}=2h, t_{up}=3h, total dist=60km, total time=5h.

Answer → average speed = 60/5 = **12 km/h**. ✅ (matches 300/25)

Mini-example — using the b, s form (b=8, s=2)

avg = $b - s^2/b$ = 8 − 4/8 = 8 − 0.5 = **7.5 km/h**. Cross-check: D=10,U=6 → 2×10×6/16 = 120/16 = 7.5. ✅

Try these

D=20,U=12 → avg=**15**

D=18,U=12 → avg=**14.4**

b=9,s=3 → avg=**8**

b=12,s=4 → avg=**10.67**

Average speed is **always less than b** (still-water speed) since $s^2/b > 0$ — a quick way to check your answer isn't too high.

⑤ Ratio & "Times as Long" Problems

For the **same distance**, time is inversely proportional to speed, so t_{up} / t_{down} = D / U. This single idea cracks every "takes twice/thrice as long" question.

The shortcut

- If $t_{up} = k \times t_{down}$ ⇒ $D = k \cdot U$ ⇒ **$b : s = (k+1) : (k-1)$**

Worked example — a boat takes thrice as long upstream as downstream

Step 1 — $k = 3$ ∴ $b{:}s = (3+1){:}(3-1) = 4{:}2$.

Step 2 — simplify $4{:}2 = 2{:}1$.

Answer → $b{:}s$ = **2:1**. Check with $b=2, s=1$: $D=3, U=1$, $d=3 \rightarrow t_{down}=1h$, $t_{up}=3h = 3{\times}t_{down}$.

✅

$k = t_{up}/t_{down}$	$b : s$	$k = t_{up}/t_{down}$	$b : s$
2	**3 : 1**	4	**5 : 3**
3	**2 : 1**	5	**3 : 2**
1.5	**5 : 1**	1.25	**9 : 1**

Derive it in one line: $b+s = k(b-s) \Rightarrow b(1-k) = -s(1+k) \Rightarrow b/s = (k+1)/(k-1)$. No need to memorise — just re-derive if you forget!

✓ "Twice as long" & "half the speed" mean the **same thing** here — both give $D = 2U$, so don't be thrown by the wording.

⑥ Same-Distance Time-Difference ⏱

A very common SSC pattern: the same distance is rowed both ways, and you're told upstream takes a certain amount of time **more** than downstream. Find the distance.

The formula

$\Delta t = t_{up} - t_{down} = d/U - d/D = \mathbf{d(D-U) / (DU) = 2ds / (DU)}$

Rearranged to find distance: $\mathbf{d = \Delta t \times D \times U / (D-U)}$

Worked example — b=10, s=2, upstream takes 1 h more

Step 1 — D = 12, U = 8.

Step 2 — d = 1 × 12 × 8 / (12−8) = 96/4.

Answer → d = **24 km**. Check: t_{down}=24/12=2h, t_{up}=24/8=3h, diff=1h. ✅

Mini-example — b=9, s=3, upstream takes 1 h more

D=12, U=6 → d = 1×12×6/(12−6) = 72/6 = **12 km**. Check: t_{down}=1h, t_{up}=2h. ✅

Try these

b=8,s=2,Δt=1h → d=**15 km**

b=11,s=3,Δt=2h → d=**33.6 km**

💡 Never subtract $t_{down} - t_{up}$ by mistake — upstream is always the **slower, longer** leg, so $\Delta t = t_{up} - t_{down}$ is positive.

✓ This is the mirror image of page 4's average-speed idea — both come from writing t_{down} and t_{up} separately and combining them.

⑦ Distance from Total Round-Trip Time

*Flip page 3's idea around: given the **total** time for a there-and-back trip, find the one-way distance.*

The formula

$T = d/D + d/U = \mathbf{d(D+U) / (DU) = 2bd / (DU)}$ (since $D+U = 2b$)

Rearranged: $\mathbf{d = T \times D \times U / (D+U) = T \times D \times U / 2b}$

Worked example — b=6, s=2, round trip takes 3 h

Step 1 — D=8, U=4.

Step 2 — $d = 3 \times 8 \times 4 / (8+4) = 96/12$.

Answer → d = **8 km**. Check: t_{down}=1h, t_{up}=2h, total=3h. ✅

Worked example — b=8, s=2, round trip takes 4 h

D=10, U=6 → d = 4×10×6/16 = 240/16 = **15 km**. Check: t_{down}=1.5h, t_{up}=2.5h, total=4h. ✅

Try these

b=9,s=3,T=4h → d=**16 km**

b=7,s=1,T=7h → d=**24 km**

Notice D + U = 2b always — it lets you swap (D+U) for 2b in any formula and work with whichever is given.

✓ Similarly D − U = 2s always — the two identities D+U=2b and D−U=2s are the engine behind every formula on pages 4, 6 & 7.

⑧ Man's Speed & Current Variations

Many questions describe b as a **multiple** of s ("rows n times as fast as the current"), or change the current mid-problem. Translate the words into $b = n \cdot s$ and reuse pages 1–7.

If $b = n \times s$

$$D = (n+1)s, \quad U = (n-1)s \quad \therefore \quad t_{up} : t_{down} = D : U = (n+1) : (n-1)$$

Worked example — a man rows 3× the speed of the current; current = 2.5 km/h

Step 1 — $b = 3 \times 2.5 = 7.5$ km/h.

Step 2 — $D = 7.5+2.5 = 10$ km/h; $U = 7.5-2.5 = 5$ km/h.

Answer → $t_{up}:t_{down} = 10:5 =$ **2:1**. ✅ (matches page 5's $k=2$ → $b:s=3:1$, since $7.5:2.5=3:1$)

Worked example — if the current's speed doubles

$b=10$, original $s=2$ → original $U=8$. New $s=4$ → new $U=$**6 km/h** — a drop of 2 km/h (25% slower) even though b never changed. ✅

Try these

$b=5s$, $s=3$ → $D=$**18**, $U=$**12**

$b=4s$, $s=4$ → $D=$**20**, $U=$**12**

"n times as fast with the stream as against it" means $D = n \cdot U$ — that's the page-5 ratio idea, **not** $b = n \cdot s$. Read carefully!

✓ Whenever b is stated as a multiple of s, plug straight into $D=(n+1)s$ and $U=(n-1)s$ — you rarely need b and s separately.

⑨ Two-Journey Simultaneous Equations

Toughest SSC type: two different mixed journeys, each with its own total time, and D, U are both unknown. Solve by treating **1/U** and **1/D** as the unknowns — the equations become linear.

The trick

Let $x = 1/U$ and $y = 1/D$. Each journey "p km up + q km down = t hours" becomes $px + qy = t$ — ordinary simultaneous equations!

Worked example — 24 km up + 28 km down in 6 h; 30 km up + 21 km down in 6.5 h

Step 1 — $24x + 28y = 6$ ...(i); $30x + 21y = 6.5$ ...(ii).

Step 2 — $30\times$(i) − $24\times$(ii): $720x+840y=180$; $720x+504y=156$ ⇒ $336y=24$ ⇒ $y=1/14$ ⇒ $D=14$.

Step 3 — sub into (i): $24x + 28/14 = 6$ ⇒ $24x = 4$ ⇒ $x = 1/6$ ⇒ $U = 6$.

Answer → $b=(14+6)/2=$**10 km/h**, $s=(14-6)/2=$**4 km/h**. Check (ii): $30/6+21/14=5+1.5=6.5$. ✅

Pick multipliers that make the x (or y) coefficients equal, exactly like eliminating a variable in normal simultaneous equations.

✓ Try one yourself → "30 up + 44 down in 10 h; 40 up + 55 down in 13 h" gives $D=11$, $U=5$, $b=8$, $s=3$ (see Exercise Q12).

⑩ Formula Master-Table

Every Boat & Stream question on pages 1–9 boils down to one of these ten lines. Scan this table first in the exam — identify what's given, then pick the matching row.

Quantity	Formula
Downstream speed	$D = b + s$
Upstream speed	$U = b - s$
Still-water speed	$b = (D+U) / 2$
Stream speed	$s = (D-U) / 2$
Time (either leg)	t = distance / speed
Avg speed (round trip)	$2DU / (D+U) = b - s^2/b$
Time ratio (same d)	$t_{up} : t_{down} = D : U$
If $t_{up}=k \cdot t_{down}$	$b : s = (k+1) : (k-1)$
Time difference (dist d)	$\Delta t = d(D-U) / (DU)$
Round-trip time (dist d)	$T = d(D+U) / (DU) = 2bd/(DU)$

Two identities behind everything

· $D + U = 2b$ · $D - U = 2s$ — use these to swap between (D,U) and (b,s) forms instantly.

Exam strategy → write down what's **given** (D, U, b, s, t or d) in the margin first, then hunt this table for the row that connects them.

✓ Units check → if speed is in km/h and time in minutes, convert one before plugging in — the single most common exam slip.

High-Frequency SSC Shortcuts

If downstream : upstream speed = a : c

Still-water : stream ratio = **$b : s = (a+c) : (a-c)$** (same idea as page 5, with D:U instead of t_{up}:t_{down})

D : U	b : s	D : U	b : s
3 : 1	**2 : 1**	5 : 3	**4 : 1**
7 : 3	**5 : 2**	9 : 5	**7 : 2**

Worked example — D : U = 5 : 3, if s = 4 km/h find b

$b{:}s = (5+3){:}(5-3) = 8{:}2 = 4{:}1 \Rightarrow b = 4 \times s = 4 \times 4 =$ **16 km/h**. ✅

More one-line shortcuts

- Boat speed is **x% more** downstream than upstream ⇒ use $D = U(1+x/100)$, then b,s from D,U.
- "Return by the same route" questions always mean **equal distance** both ways — page 4 & 7 formulas apply directly.
- If a swimmer's/boat's speed in still water is asked and only D, U are given, it's **always** $b = (D+U)/2$ — no shortcuts needed, just plug in.

Ratio questions are the fastest-scoring type in this chapter — if you can spot "a:c" or "k times" language, the answer is two steps away.

Practice reading the question **twice** before solving — boat & stream traps usually hide in whether a ratio refers to speed or to time.

Solved Examples – Set I

Q1. A man rows at 6 km/h in still water; the river flows at 2 km/h. Find the time to row to a place 16 km away and back.
$D=8,\ U=4 \rightarrow t_{down}=16/8=2h,\ t_{up}=16/4=4h \therefore$ total = **6 hours**. ✅

Q2. A boat's downstream speed is 24 km/h and upstream speed is 16 km/h. Find b and s.
$b=(24+16)/2=$**20 km/h**; $s=(24-16)/2=$**4 km/h**. ✅

Q3. A boat covers 45 km downstream in 3 h and returns upstream in 5 h. Find b and s.
$D=45/3=15,\ U=45/5=9 \rightarrow b=(15+9)/2=$**12 km/h**; $s=(15-9)/2=$**3 km/h**. ✅

Q4. Find the average speed for a round trip if a boat goes downstream at 15 km/h and returns upstream at 10 km/h.
avg $= 2\times15\times10/(15+10) = 300/25 =$ **12 km/h**. ✅

Q5. A man can row 3 times as fast with the stream as against it. The stream's speed is 4 km/h. Find his speed in still water.
$D=3U \rightarrow b+s=3(b-s) \Rightarrow 4s=2b \Rightarrow b=2s = 2\times4 =$ **8 km/h**. Check: $D=12, U=4, D=3U$. ✅

Q6. A boat's speed in still water is 5 times the stream's speed. It covers 24 km downstream in 2 h. Find the stream's speed.
$D=24/2=12;\ b=5s \Rightarrow D=b+s=6s=12 \Rightarrow s=$**2 km/h** $(b=10)$. Check: $U=8$. ✅

✓ Notice Q1–Q6 only ever use four ideas: $D=b+s$, $U=b-s$, $t=d/\text{speed}$, and avg speed = $2DU/(D+U)$. Master these four and this chapter is done.

Solved Examples – Set II

Q7. $b=9$ km/h, $s=3$ km/h. Upstream takes 1 h longer than downstream for the same distance. Find the distance.

$D=12, U=6 \rightarrow d = 1\times12\times6/(12-6) = 72/6 =$ **12 km**. Check: $t_{down}=1h$, $t_{up}=2h$. ✅

Q8. $b=12$ km/h, $s=4$ km/h. Find total time for a round trip to a place 32 km away and back.

$D=16, U=8 \rightarrow t_{down}=32/16=2h$, $t_{up}=32/8=4h$ ∴ total = **6 hours**. ✅

Q9. A man rows 12 km upstream in 3 h and 12 km downstream in 2 h. Find b and s.

$U=12/3=4$, $D=12/2=6 \rightarrow b=(6+4)/2=$**5 km/h**; $s=(6-4)/2=$**1 km/h**. ✅

Q10. A boat's downstream speed is 5/3 times its upstream speed. Its still-water speed is 8 km/h. Find the stream's speed.

$D=(5/3)U \Rightarrow 3(b+s)=5(b-s) \Rightarrow 8s=2b \Rightarrow b=4s \Rightarrow s = 8/4 =$ **2 km/h**. Check: $D=10, U=6$, $D/U=5/3$. ✅

Q11. $b=10$ km/h, $s=2$ km/h originally. If the current's speed doubles, find the new upstream speed.

New $s=4 \rightarrow$ new $U = 10-4 =$ **6 km/h** (down from the original 8 km/h). ✅

Q12. A man's speed in still water is thrice the stream's speed. He rows 24 km downstream in 2 h. How far can he row upstream in the same 2 h?

$D=24/2=12=4s \Rightarrow s=3, b=9 \Rightarrow U=b-s=6 \Rightarrow$ distance in 2h $= 6\times2 =$ **12 km**. ✅

💡 Q11 & Q12 show the two most-tested twists — a mid-question change in current, and a "same time, different distance" comparison. Both reduce to plain $D=b+s$, $U=b-s$.

Exercise

12 MCQs covering every idea from pages 1–11. Answer key with working is on the next page.

1. Boat's speed in still water = 12 km/h, stream = 3 km/h. Downstream speed = ?
(a) 9 (b) 15 (c) 12 (d) 36

2. Using Q1's data, the upstream speed = ?
(a) 15 (b) 9 (c) 3 (d) 4

3. Downstream speed = 20 km/h, upstream = 10 km/h. Speed in still water = ?
(a) 10 (b) 15 (c) 5 (d) 30

4. Using Q3's data, the speed of the stream = ?
(a) 5 (b) 10 (c) 15 (d) 30

5. A boat covers 36 km downstream in 3 h; stream = 4 km/h. Still-water speed = ?
(a) 8 (b) 12 (c) 16 (d) 4

6. b=15 km/h, s=5 km/h. Total time to row 40 km upstream and return = ?
(a) 4h (b) 5h (c) 6h (d) 8h

7. D=18 km/h, U=12 km/h. Average round-trip speed = ?
(a) 15 (b) 14.4 (c) 14 (d) 16

8. A boat takes twice as long upstream as downstream (same distance). Ratio b:s = ?
(a) 2:1 (b) 3:1 (c) 3:2 (d) 4:1

9. b=9, s=3. Upstream takes 2 h more than downstream for a distance. Distance = ?
(a) 12 (b) 18 (c) 24 (d) 30

10. b=8, s=2. Round trip to a place and back takes 4 h. One-way distance = ?
(a) 10 (b) 12 (c) 15 (d) 20

11. A man rows at 4× the speed of the current; current = 3 km/h. Ratio $t_{up}:t_{down}$ = ?
(a) 5:3 (b) 3:5 (c) 4:3 (d) 3:4

12. A boat covers 30km up + 44km down in 10h, and 40km up + 55km down in 13h. Still-water speed = ?
(a) 6 (b) 7 (c) 8 (d) 9

*Attempt every question **before** peeking at the answer key on the next page — timing yourself is half the practice!*

Answer Key — with working

1→ b) 15 (12+3) · **2→ b) 9** (12−3) · **3→ b) 15** ((20+10)/2) · **4→ a) 5** ((20−10)/2) · **5→ a) 8** (36/3−4) · **6→ c) 6h** (D=20,U=10: 2+4) · **7→ b) 14.4** (2×18×12/30) · **8→ b) 3:1** ((2+1):(2−1)) · **9→ c) 24** (D=12,U=6: 2×12×6/6) · **10→ c) 15** (D=10,U=6: 4×10×6/16) · **11→ a) 5:3** (b=12,D=15,U=9) · **12→ c) 8** (D=11,U=5 → b=8).

Bonus Challenge Examples

B1. A man rows to a place 48 km away and back in 14 h. He can row 4 km with the stream in the same time as 3 km against it. Find the stream's speed.

Step 1 — same time ⇒ D:U = 4:3, so let D=4k, U=3k.

Step 2 — $T = d(D+U)/(DU) \Rightarrow 14 = 48 \times 7k \ / \ 12k^2 = 28/k \Rightarrow k = 2$.

Answer → D=8, U=6 ⇒ s=(8−6)/2 = **1 km/h**. Check: 48/8+48/6=6+8=14h. ✅

B2. b = 4×s. The boat takes 6 h to cover 24 km upstream. Find the total round-trip time for the same 24 km.

Step 1 — U = b−s = 3s = 24/6 = 4 ⇒ s = 4/3, b = 16/3.

Step 2 — D = b+s = 20/3 ⇒ t_{down} = 24/(20/3) = 3.6h.

Answer → total = 6 + 3.6 = **9.6 h** (9 h 36 min). ✅

✓ *Both bonus questions mix two chapters (ratio + round-trip time) — exactly the style SSC uses for the last, toughest question in a set.*

Quick Revision 💡

- ✓ Downstream $D=b+s$, Upstream $U=b-s$. Reverse: $b=(D+U)/2$, $s=(D-U)/2$. Always $s < b$.
- ✓ Avg speed for equal-distance round trip $= 2DU/(D+U) = b-s^2/b$ — the harmonic mean, NOT $(D+U)/2$.
- ✓ $t_{up}:t_{down} = D:U$ (same distance). If $t_{up}=k\cdot t_{down}$, then $b:s = (k+1):(k-1)$.
- ✓ Time difference $\Delta t = d(D-U)/(DU)$; round-trip time $T = d(D+U)/(DU) = 2bd/(DU)$. Also $D+U=2b$, $D-U=2s$.
- ✓ Two-journey questions → let $x=1/U$, $y=1/D$ and solve as simultaneous equations.

Quick-fire recap

$D=18, U=12 \rightarrow b=$**15**, $s=$**3**

$b=10, s=4 \rightarrow avg=$**8.4 km/h**

$t_{up}=1.5\times t_{down} \rightarrow b:s=$**5:1**

$b=7, s=1, d=24 \rightarrow T=$**7 h**

One last worked example — b=13 km/h, s=3 km/h, round trip to a 40 km spot

$D=16$, $U=10 \rightarrow t_{down}=40/16=2.5h$, $t_{up}=40/10=4h \rightarrow$ total = **6.5 h**; avg speed $= 80/6.5 \approx$ **12.3 km/h**. ✅

☆ Downstream or upstream — you've got the current figured out! ☆