Average

~ Quantitative Aptitude · SSC Exams ~

← circled = key formulas!

Average finds the one representative value of a whole group. Master the **basic formula, the special series, and the add·remove·replace tricks** — almost every SSC question is built on these.

① Definition & Basic Formula

The one formula that runs the whole chapter

- **Average (A)** = Sum of all observations / Number of observations
- **Sum** = Average × Number of observations
- **Number** = Sum / Average

Worked example — runs scored in 5 innings

Scores: 45, 60, 38, 72, 50.

Step 1 — Sum = 45+60+38+72+50 = 265.

Step 2 — Number of innings = 5.

Answer → Average = 265/5 = **53**. ✅

Two quick facts examiners love

- The average always lies **between the smallest and largest** value in the data — a fast sanity check on your answer.
- If every observation equals **k**, then the average is also **k** (no spread at all).

Trap → "Average of averages" is only valid when every group has the **same size**. If group sizes differ, always go back to total sum ÷ total count (see page 9).

② Averages of Number Series

These are the ready-made formulas SSC papers test again and again — learn them cold.

Series	Average Formula	Quick example
First n natural numbers	**$(n+1)/2$**	$n=20 \rightarrow 10.5$
Consecutive numbers / AP	**(first+last)/2**	$15–45 \rightarrow 30$
First n odd numbers	**n**	$n=15 \rightarrow 15$
First n even numbers	**$n+1$**	$n=12 \rightarrow 13$

Worked example — average of numbers from 15 to 45

Step 1 — This is a consecutive (AP) series, so use (first+last)/2.

Step 2 — $(15+45)/2 = 60/2$.

Answer → Average = **30**. ✅ (Total numbers = 31, but count is not even needed here!)

Try these

- Average of first 20 natural numbers = $(20+1)/2$ = **10.5**
- Average of first 15 odd numbers = **15** · Average of first 12 even numbers = **13**
- Average of even numbers from 10 to 30 = $(10+30)/2$ = **20**

For **any** AP (numbers, odd, even, multiples of k) the average is simply **(first term + last term)/2** — you never even need to count the terms!

③ Averages of Squares & Cubes ▦

Learn these two by heart

- Average of squares of first n natural numbers = $\mathbf{(n+1)(2n+1)\,/\,6}$
- Average of cubes of first n natural numbers = $\mathbf{n(n+1)^2\,/\,4}$

These come straight from the sum formulas: $\Sigma n^2 = n(n+1)(2n+1)/6$ and $\Sigma n^3 = [n(n+1)/2]^2$ — just divide each by n.

Worked example — average of squares of first 10 natural numbers

Step 1 — use $(n+1)(2n+1)/6$ with $n = 10$.

Step 2 — $(11)(21)/6 = 231/6$.

Answer → = **38.5**. ✅

Worked example — average of cubes of first 6 natural numbers

Step 1 — use $n(n+1)^2/4$ with $n = 6$.

Step 2 — $6 \times 7^2 \,/\, 4 = 6\times49/4 = 294/4$.

Answer → = **73.5**. ✅

Fast recall — small n

Avg of squares of first 4 = **7.5**

Avg of squares of first 6 = **$15.1\overline{6}$**

Avg of cubes of first 3 = **12**

Avg of cubes of first 5 = **45**

💡 *Don't confuse "average of squares" with the "square of the average" — they are almost never equal! Check: (avg of 1,2,3)^2 = 4, but avg of squares = 14/3 ≈ 4.67.*

④ Weighted Average ⚖

Used when groups of **different sizes** are mixed together — bigger groups pull the average closer to themselves.

Formula

Weighted average = $(w_1x_1 + w_2x_2 + \ldots) / (w_1 + w_2 + \ldots)$

where w = weight (size/quantity) and x = the value for that group.

Worked example — average weight of a class

30 boys, average weight 45 kg; 20 girls, average weight 40 kg.

Step 1 — total weight = 30×45 + 20×40 = 1350+800 = 2150 kg.

Step 2 — total students = 30+20 = 50.

Answer → average = 2150 / 50 = **43 kg**. ✅

Try these

- 20 kg rice at ₹40 / kg mixed with 30 kg rice at ₹60 / kg → avg price = (800+1800) / 50 = **₹52 / kg**
- 4 subjects of 100 marks & 1 subject of 50 marks, all scored 80% → weighted avg is still **80%** (weights don't matter when % is equal!)

Simple average of 45 & 40 would wrongly give 42.5. Because there are **more boys** (heavier weight), the true average 43 leans toward 45 — always weight by group size.

Weighted average always lies **between the two group values** (here, between 40 and 45) — a handy elimination trick in MCQs.

⑤ Average Speed

Golden rule — never just average the speeds!

- General case → **Average speed = Total distance / Total time**
- **Same distance**, two speeds x & y → Average speed = **2xy / (x+y)** (harmonic mean)
- **Same distance**, three speeds x,y,z → Average speed = **3xyz / (xy+yz+zx)**
- **Same time** at different speeds → Average speed = simple average (arithmetic mean) of the speeds

Worked example — goes at 40 km/h, returns at 60 km/h

Same distance both ways (equal distance case).

Step 1 — use 2xy/(x+y) with x=40, y=60.

Step 2 — 2×40×60/(100) = 4800/100.

Answer → average speed = **48 km/h**. ✅ (Not 50 — that's the common trap!)

Mini-example — equal-time case

Drives for 2 hrs each at 40, 60 & 50 km/h (same time, not same distance) → average = (40+60+50)/3 = **50 km/h**. Check: distance = 80+120+100 = 300 km in 6 hrs = 50 km/h. ✅

Ask yourself first — is the **distance** equal or the **time** equal? Equal distance ⇒ harmonic-mean formula. Equal time ⇒ plain arithmetic mean.

x=20,y=30 (equal dist) → **24 km/h**

x=y (same speed both ways) → avg = **x** itself

⑥ Effect of Including a New Member

Formula

- New average = (Old sum + New value) / (Old count + 1)
- Value of new member = New average × (n+1) − Old average × n

Worked example — teacher joins the class

Average age of 24 students is 15 years. Including the teacher, average becomes 16 years.

Step 1 — New sum (25 people) = 16 × 25 = 400.

Step 2 — Old sum (24 students) = 15 × 24 = 360.

Answer → Teacher's age = 400 − 360 = **40 years**. ✅

Try these

- Average of 19 numbers is 40. A new number joins, avg becomes 41 → new number = 41×20−40×19 = 820−760 = **60**
- Average weight of 9 boys is 50 kg. A 10th boy joins and average falls to 49 kg → his weight = 49×10−50×9 = 490−450 = **40 kg**

*If the new value is **above** the old average, the average **rises**; if it's **below**, the average **falls**. Check the direction before calculating — it catches silly sign errors.*

✓ *Shortcut → New member's value = New average + n × (change in average). In the teacher example: 16 + 24×1 = **40**. Same answer, one line!*

⑦ Effect of Excluding a Member

Formula

- New average = (Old sum − Removed value) / (Old count − 1)
- Value removed = Old average × n − New average × (n−1)

Worked example — one person leaves the group

Average weight of 8 persons is 65 kg. One person leaves, average of remaining 7 becomes 63 kg.

Step 1 — Old sum = 65 × 8 = 520.

Step 2 — New sum = 63 × 7 = 441.

Answer → Weight of the person who left = 520 − 441 = **79 kg**. ✅

Try these

- Average of 9 numbers is 25. Highest number is removed, average of remaining 8 becomes 23 → highest = 225−184 = **41**
- Average of 6 numbers is 30. A number 45 is removed → new average = (180−45)/5 = 135/5 = **27**

If removing a value **raises** the average, that value was **below** the old average; if it **lowers** the average, the removed value was **above** it — opposite logic to adding a member!

Shortcut → Removed value = Old average − n × (change in average). Here: 65 − 8×(−2) = 65+16 = **81**? Careful with signs — always verify with the direct sum method above.

⑧ Replacement Problems ⇄

Classic "when a person is replaced by another..." age/weight questions — one formula solves them all.

Formula

New value − Old value = n × (change in average)

Use +change if average increased, −change if average decreased. n = number of members (unchanged by a swap).

Worked example — a member is swapped

Average age of 10 members increases by 2 years when a member aged 18 is replaced by a new member.

Step 1 — New − Old = n × change = 10 × 2 = 20.

Step 2 — New = Old + 20 = 18 + 20.

Answer → New member's age = **38 years**. ✅

Quick reference — all three effects together

Action	Formula
Adding a member	New avg = (Old sum + new value) / (n+1)
Removing a member	Value removed = Old avg×n − New avg×(n−1)
Replacing a member	New value − Old value = n × change in avg

💡 In a replacement, the group size **never changes** — only the total sum shifts by exactly (New−Old). That single fact powers the whole formula.

⑨ Combining Two or More Groups

Formula

Combined average = $(n_1A_1 + n_2A_2 + \dots) / (n_1 + n_2 + \dots)$

This is just the weighted-average idea (page 4) applied to whole groups instead of single values.

Worked example — two sections combined

Section A: 30 students, average marks 70. Section B: 20 students, average marks 80.

Step 1 — total marks = 30×70 + 20×80 = 2100+1600 = 3700.

Step 2 — total students = 50.

Answer → combined average = 3700 / 50 = **74**. ✅

Mini-example — three groups

10 items avg 20, 15 items avg 30, 25 items avg 40 → combined = (200+450+1000) / 50 = 1650 / 50 = **33**. ✅

Reverse-use of the formula

If you know the combined average and one group's average, you can find the **other group's average** or even the **ratio n_1:n_2** by cross-multiplying — a favourite twist in SSC.

💡 *Never average the two group-averages directly (70 & 80 ≠ 75 here) unless $n_1 = n_2$. Always weight by group size.*

⑩ Problems on Innings / Scores

A batsman's "innings" problem is really the **add-a-new-member** idea (page 6) dressed up in cricket language.

Worked example — average rises after the 13th innings

A cricketer's average after 12 innings is 40 runs. He scores 92 in the 13th innings.

Step 1 — old sum = 40 × 12 = 480.

Step 2 — new sum = 480 + 92 = 572.

Answer → new average = 572 / 13 = **44**. ✅

Worked example — excluding highest & lowest scores

Average of 40 innings is 50. Highest exceeds lowest by 172. Excluding both, average of remaining 38 innings is 48.

Step 1 — sum of 40 = 40×50 = 2000; sum of 38 = 38×48 = 1824.

Step 2 — highest + lowest = 2000−1824 = 176; highest − lowest = 172.

Answer → highest = (176+172) / 2 = **174**, lowest = **2**. ✅

Try these

- Average after 15 innings = 32. Scores 92 in 16th → new avg = (480+92) / 16 = **35.75**
- To raise average from 30 to 32 in the 11th innings (10 played so far), runs needed = 32×11−30×10 = 352−300 = **52**

✓ "Excluding the highest and lowest" questions always reduce to two equations: (H+L) from the sum-difference, and (H−L) from the given gap — solve together.

⑪ Average Age & Family Problems

Key idea

- If **no one** joins or leaves, after t years every member is t years older, so the **average also rises by exactly t**.
- If someone is **born** or **leaves** during that time, adjust the total separately before dividing.

Worked example — age at the birth of the youngest

Average age of a family of 6 is 22 years; youngest member is 7 years old. Find the family's average age at the youngest's birth (i.e. 7 years ago, excluding the youngest).

Step 1 — present total age = $6 \times 22 = 132$; other 5 members' present total = $132 - 7 = 125$.

Step 2 — 7 years ago, their total was $125 - 5 \times 7 = 125 - 35 = 90$.

Answer → average then = $90 / 5$ = **18 years**. ✅

Worked example — a third person joins later

Average age of A & B, 5 years ago, was 15 years. Average age of A, B & C today is 20 years. Find C's age after 5 more years.

Step 1 — A+B total 5 yrs ago = 30 → today = $30 + 2 \times 5 = 40$.

Step 2 — A+B+C today = $20 \times 3 = 60$ → C's age today = $60 - 40 = 20$.

Answer → C's age after 5 years = $20 + 5$ = **25 years**. ✅

*Always ask "how many people were actually present at that time?" before dividing — the youngest member **didn't exist** 7 years ago, so the count drops to 5, not 6.*

⑫ Shortcut – Assumed-Average Method ◎

Formula

Average = Assumed average (a) + (Sum of deviations from a) / n

deviation = each value − a. Picking a value near the "middle" of the data keeps the deviations small & the mental math fast.

Worked example – find the average fast

Data: 42, 45, 49, 53, 56. Take assumed average a = 50.

Step 1 — deviations: −8, −5, −1, +3, +6 → sum = −5.

Step 2 — average = 50 + (−5 / 5) = 50 − 1.

Answer → average = **49**. ✅ (Direct sum / 5 also gives 245 / 5 = 49 — confirmed.)

Mini-example – deviations cancel exactly

78, 82, 69, 91, 75, 85 with a = 80 → deviations −2,+2,−11,+11,−5,+5 sum to **0** → average = **80** instantly — no division needed! ✅

Pick a round-number assumed average that's **close to the middle** of the data – it makes the deviations small, positive & negative deviations often cancel, and mental addition stays quick.

✓ This is exactly how surveyors & exam-solvers find averages of ugly numbers (like marks out of 100 close to 70–90) without long division.

Solved Examples – I

Q1. Average of 11 results is 50. Average of first 6 is 49, of last 6 is 52. Find the 6th result.
Sum(1–6)+Sum(6–11) = Sum(1–11) + 6th result (counted twice) → 294+312 = 550+6th → 6th = **56**. ✅

Q2. 20 kg rice at ₹40/kg mixed with 30 kg rice at ₹60/kg. Find average price per kg.
$(20\times40+30\times60)/50 = (800+1800)/50 = 2600/50 =$ **₹52**. ✅

Q3. A man covers a distance going at 20 km/h and returns at 30 km/h. Find his average speed.
$2xy/(x+y) = 2\times20\times30/50 = 1200/50 =$ **24 km/h**. ✅

Q4. Average weight of 6 men increases by 1.5 kg when a 65 kg man is replaced by a new man. Find the new man's weight.
New = Old + n×change = $65 + 6\times1.5 = 65+9 =$ **74 kg**. ✅

Q5. Average marks of 30 boys is 60 and of 20 girls is 70. Find average of all 50 students.
$(30\times60+20\times70)/50 = (1800+1400)/50 = 3200/50 =$ **64**. ✅

Q6. Find the average of squares of the first 5 natural numbers.
$(n+1)(2n+1)/6 = 6\times11/6 =$ **11**. ✅

✓ Notice how every question reduces to **sum = average × count** somewhere in the working – that's the one idea to hold onto under exam pressure.

Solved Examples – II

Q7. Average age of 30 kids in a class is 12 years. Average age of 20 of them is 11 years. Find the average age of the remaining 10.

Sum30–Sum20 = 360–220 = 140 → average of remaining 10 = 140/10 = **14 years**. ✅

Q8. A cricketer's average after 12 innings is 40 runs. He scores 92 in the 13th innings. Find his new average.

New sum = 480+92 = 572 → new average = 572/13 = **44**. ✅

Q9. Using the assumed-average method (a=80), find the average of 81, 79, 85, 77, 83.

Deviations: +1, −1, +5, −3, +3 → sum=5 → average = 80+5/5 = **81**. ✅

Q10. Average of 9 numbers is 25. Excluding the highest, average of remaining 8 is 23. Find the highest number.

Sum9–Sum8 = 225–184 = **41**. ✅

Q11. A car travels 2 hours each at 40 km/h, 60 km/h and 50 km/h. Find its average speed.

Equal time → arithmetic mean = (40+60+50)/3 = **50 km/h**. Check: 300 km/6 h = 50 km/h. ✅

Speed questions almost always reveal themselves by whether the phrase is "covers a distance" (equal distance) or "travels for … hours" (equal time) – read carefully before picking the formula.

Exercise

1. The average of the first 30 natural numbers is:
(a) 15 (b) 15.5 (c) 16 (d) 14.5

2. Average of all odd numbers from 1 to 21 is:
(a) 10 (b) 11 (c) 12 (d) 21

3. Average of squares of first 4 natural numbers is:
(a) 7 (b) 7.5 (c) 8 (d) 6.5

4. Average of cubes of first 3 natural numbers is:
(a) 10 (b) 12 (c) 14 (d) 9

5. Average marks of 19 students is 40. A new student scoring 60 joins. New average of 20 students?
(a) 40 (b) 41 (c) 42 (d) 45

6. Average of 8 numbers is 20. One number is removed, average becomes 18. Removed number?
(a) 30 (b) 32 (c) 34 (d) 36

7. Average age of 15 members increases by 2 yrs when a 20-year-old is replaced. New person's age?
(a) 40 (b) 45 (c) 50 (d) 55

8. Average marks of 30 boys is 60 and of 20 girls is 70. Combined average of 50 students?
(a) 62 (b) 63 (c) 64 (d) 65

9. A man travels equal distances at 15 km/h and 25 km/h. Average speed?
(a) 18.75 (b) 20 (c) 19.5 (d) 20.5

10. Cricketer's average after 10 innings is 32. Scores 76 in 11th. New average?
(a) 34 (b) 35 (c) 36 (d) 38

11. Average of 9 consecutive numbers is 20. The largest number is:
(a) 22 (b) 23 (c) 24 (d) 25

12. Assumed avg = 50; deviations of 5 numbers: +4,−2,+6,−3,−5. Actual average?
(a) 48 (b) 49 (c) 50 (d) 51

Solve on paper first, then check against the Answer Key on the next page — genuine practice beats reading formulas twice!

Answer Key — with working

1→ (b) 15.5 ($(n+1)/2$) · **2→ (b) 11** (avg of first n odd = n) · **3→ (b) 7.5** ($5\times9/6$) · **4→ (b) 12** ($3\times16/4$) · **5→ (b) 41** ($(760+60)/20$) · **6→ (c) 34** ($160-126$) · **7→ (c) 50** ($20+15\times2$) · **8→ (c) 64** ($(1800+1400)/50$) · **9→ (a) 18.75** ($2\times15\times25/40$) · **10→ (c) 36** ($(320+76)/11$) · **11→ (c) 24** (20 is middle of 16–24) · **12→ (c) 50** (deviations sum to 0).

Bonus Solved Examples

Bonus 1. Average of 5 consecutive even numbers is 24. Find the largest number.
Middle (3rd) number = 24 → the five numbers are 20,22,24,26,28 → largest = **28**. ✅

Bonus 2. Average of 3 numbers is 42. First is twice the second, second is twice the third. Find them.
Let third = x, second = $2x$, first = $4x$ → sum = $7x = 126$ → $x = 18$. So numbers are **72, 36, 18**. ✅

Quick Revision

- ✓ Average = Sum/Count. First n naturals=$(n+1)/2$; AP=(first+last)/2; first n odd=n; first n even=n+1; squares=$(n+1)(2n+1)/6$; cubes=$n(n+1)^2/4$.
- ✓ Weighted & combined-group average = $\Sigma(n\times A)/\Sigma n$. Equal-distance speed → $2xy/(x+y)$; equal-time speed → arithmetic mean.
- ✓ Add/remove/replace — always convert to sums first. Replacement: New−Old = n×change. Assumed-average shortcut: $A = a + \Sigma\text{deviations}/n$.

☆ Average — sum divided into equal shares! ☆