Algebra

~ Quantitative Aptitude · SSC Exams ~

← circled = key info!

Algebra is a game of identities. Learn each formula, then **see it inside** the question. Most SSC sums are one identity in disguise.

① Basic (Square) Identities

Memorise these five — the backbone of the chapter

- $(a+b)^2 = a^2 + 2ab + b^2$ ex: $(x+5)^2 = x^2+10x+25$
- $(a-b)^2 = a^2 - 2ab + b^2$ ex: $(2x-3)^2 = 4x^2-12x+9$
- $a^2 - b^2 = (a+b)(a-b)$ ex: $53^2-47^2 = 100\times6 = 600$
- $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$ ex: $a=3, b=1 \rightarrow 16+4 = 2(10)$
- $(a+b)^2 - (a-b)^2 = 4ab$ ex: $a=3, b=1 \rightarrow 16-4 = 4\times3$

Worked example — find 105^2 without long multiplication

Step 1 — write $105 = 100 + 5$, use $(a+b)^2$ with $a=100$, $b=5$.

Step 2 — $100^2 + 2\times100\times5 + 5^2 = 10000 + 1000 + 25$.

Answer → 105^2 = **11025**. ✅ ($96^2 = (100-4)^2 = 10000-800+16 = 9216$)

✓ The gap-of-squares trick: $a^2-b^2 = (a+b)(a-b)$. So $51^2-49^2 = 100\times2 =$ **200** in one line.

Try these

$(x+7)^2 =$ **$x^2+14x+49$**

$(3a-2)^2 =$ **$9a^2-12a+4$**

$82^2-18^2 = 100\times64 =$ **6400**

if $a+b=6$, $ab=8 \rightarrow a^2+b^2 = 36-16 =$ **20**

Key rearrangement (used everywhere)

$a^2+b^2 = (a+b)^2 - 2ab = (a-b)^2 + 2ab$ · $(a-b)^2 = (a+b)^2 - 4ab$.

Working with a+b, a−b & ab

Most identity questions give you **two** of $\{a+b, a-b, ab, a^2+b^2\}$ and ask a third. This triangle of relations links them all.

The bridge formulas

- $(a+b)^2 = a^2+b^2 + 2ab$
- $(a-b)^2 = a^2+b^2 - 2ab$
- $(a+b)^2 - (a-b)^2 = 4ab \therefore ab = [(a+b)^2-(a-b)^2]/4$
- $a^2+b^2 = ½[(a+b)^2 + (a-b)^2]$

Worked example — $a+b = 9$ and $ab = 20$, find a^2+b^2 and $a-b$

Step 1 — $a^2+b^2 = (a+b)^2 - 2ab = 81 - 40 =$ **41**.

Step 2 — $(a-b)^2 = (a+b)^2 - 4ab = 81 - 80 = 1$.

Answer → $a-b =$ **±1** (the numbers are 4 & 5). ✅

$a^2 - b^2$ in fast arithmetic

Worked — evaluate $(2.7)^2 - (1.3)^2$

$= (2.7+1.3)(2.7-1.3) = 4.0 \times 1.4 =$ **5.6**. One multiplication instead of two squares.

Spot the pattern: if you see a **difference of two squares**, never expand — factor to (sum)×(difference) and the answer falls out.

More two-value drills

- $a+b = 10$, $a^2+b^2 = 58 \rightarrow 2ab = 100-58 = 42 \rightarrow ab =$ **21**.
- $a-b = 4$, $ab = 12 \rightarrow a^2+b^2 = 16+24 =$ **40**; $(a+b)^2 = 40+24 = 64 \rightarrow a+b =$ **8**.
- $a+b = 7$, $a-b = 3 \rightarrow ab = (49-9)/4 =$ **10**; $a=5$, $b=2$.

✓ Golden identity of this page → $(a+b)^2 = (a-b)^2 + 4ab$. Whenever ab is known, one square gives the other.

② Cubic Identities

The four cube formulas

- $(a+b)^3 = a^3 + b^3 + 3ab(a+b) = a^3+3a^2b+3ab^2+b^3$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b) = a^3-3a^2b+3ab^2-b^3$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(x+2)^3 = x^3+6x^2+12x+8$

$(x-1)^3 = x^3-3x^2+3x-1$

$7^3+3^3 = 10(49-21+9) =$ **370**

$6^3-4^3 = 2(36+24+16) =$ **152**

Worked example — $a+b = 5$, $ab = 6$, find a^3+b^3

Step 1 — use $a^3+b^3 = (a+b)^3 - 3ab(a+b)$.

Step 2 — $= 5^3 - 3\times6\times5 = 125 - 90$.

Answer → $a^3+b^3 =$ **35** ($a=2$, $b=3 \to 8+27 = 35$ ✓). ✅

Rewrite forms → $a^3+b^3 = (a+b)^3 - 3ab(a+b)$ and $a^3-b^3 = (a-b)^3 + 3ab(a-b)$. These are the exam-favourite versions.

Worked example — $a-b = 3$, $ab = 10$, find a^3-b^3

Step 1 — $a^3-b^3 = (a-b)^3 + 3ab(a-b) = 3^3 + 3\times10\times3$.

Answer → $27 + 90 =$ **117** ($a=5$, $b=2 \to 125-8 = 117$ ✓). ✅

Try these

$a+b=4$, $ab=3 \to a^3+b^3 = 64-36 =$ **28**

$a-b=2$, $ab=15 \to a^3-b^3 = 8+90 =$ **98**

Sum & Difference of Cubes — applied

The factor forms turn ugly cubes into a product you can cancel or evaluate. Watch how the quadratic factor $\mathbf{a^2 \pm ab + b^2}$ appears.

Worked example — simplify $(a^3+b^3)/(a+b)$

Step 1 — $a^3+b^3 = (a+b)(a^2-ab+b^2)$.

Step 2 — cancel the common $(a+b)$.

Answer → $= a^2 - ab + b^2$. ✓

Worked example — if $a^2+b^2 = 29$ and $ab = 10$, find a^3+b^3

Step 1 — $(a+b)^2 = a^2+b^2+2ab = 29+20 = 49 \rightarrow a+b = 7$.

Step 2 — $a^2-ab+b^2 = 29 - 10 = 19$.

Step 3 — $a^3+b^3 = (a+b)(a^2-ab+b^2) = 7\times19$.

Answer → **133**. ✓

✓ *Handy: $a^2-ab+b^2 = (a+b)^2 - 3ab$, and $a^2+ab+b^2 = (a-b)^2 + 3ab$. Everything reduces to (sum/diff) & ab.*

Numeric shortcuts

- $12^3 + 8^3 = (20)(144-96+64) = 20\times112 =$ **2240**.
- $15^3 - 5^3 = (10)(225+75+25) = 10\times325 =$ **3250**.
- $(101)^3 = (100+1)^3 = 10^6 + 3(100^2) + 3(100) + 1 =$ **1030301**.

If a^3+b^3 is asked and only a+b & ab are given, do NOT find a,b — feed straight into $(a+b)^3-3ab(a+b)$.

Try these

$a+b=6,\ ab=5 \rightarrow a^3+b^3 = 216-90 =$ **126**

$a^2+b^2=13,\ ab=6 \rightarrow a^3+b^3 = 5\times7 =$ **35**

③ Three-Variable Identities △

The two master formulas

- $(a+b+c)^2 = a^2+b^2+c^2 + 2(ab+bc+ca)$
- $a^3+b^3+c^3 - 3abc = (a+b+c)(a^2+b^2+c^2 - ab-bc-ca)$

$\therefore a^2+b^2+c^2 = (a+b+c)^2 - 2(ab+bc+ca)$

The BIG special case ⟶ **IF $a+b+c = 0$** THEN $a^3+b^3+c^3 =$ **$3abc$** and $a^2+b^2+c^2 =$ **$-2(ab+bc+ca)$**. Extremely high-frequency in SSC.

Worked — $a+b+c = 6$, $ab+bc+ca = 11$, find $a^2+b^2+c^2$

Step 1 — $a^2+b^2+c^2 = (a+b+c)^2 - 2(ab+bc+ca)$.

Step 2 — $= 6^2 - 2\times11 = 36 - 22$.

Answer → **14**. ✅

Worked — if $a+b+c = 0$, evaluate $a^3+b^3+c^3$ for 2, −3, 1

Check — $2 + (-3) + 1 = 0$ ∴ use $a^3+b^3+c^3 = 3abc$.

Compute — $3\times2\times(-3)\times1 =$ **−18**. $(8-27+1 = -18$ ✓$)$ ✅

Classic exam shape

If $x = a-b$, $y = b-c$, $z = c-a$ then $x+y+z = 0$, so **$x^3+y^3+z^3 = 3xyz$** $= 3(a-b)(b-c)(c-a)$. This one is asked again and again.

Try these

$a+b+c=9$, $ab+bc+ca=23$ → $\Sigma a^2 = 81-46 =$ **35**

$3+4+(-7)=0$ → $\Sigma a^3 = 3(3)(4)(-7) =$ **−252**

④ The $x + 1/x$ Family X −

The single most-tested idea in SSC algebra. Everything is powered by squaring / cubing $\mathbf{x + 1/x}$.

If $x + 1/x = k$ (the "plus" family)

- $x^2 + 1/x^2 = \mathbf{k^2 - 2}$
- $x^3 + 1/x^3 = \mathbf{k^3 - 3k}$
- $x^4 + 1/x^4 = \mathbf{(k^2-2)^2 - 2}$

why? $(x + 1/x)^2 = x^2 + 2 + 1/x^2$, and $(x + 1/x)^3 = x^3 + 1/x^3 + 3(x + 1/x)$

Ready results table — the "plus" family

$x + 1/x = k$	Result formula	$k = 3$ gives
x^2+1/x^2	$k^2 - 2$	$9 - 2 = \mathbf{7}$
x^3+1/x^3	$k^3 - 3k$	$27 - 9 = \mathbf{18}$
x^4+1/x^4	$(k^2-2)^2 - 2$	$7^2 - 2 = \mathbf{47}$
x^5+1/x^5	$(x^2+1/x^2)(x^3+1/x^3) - k$	$7\times18 - 3 = \mathbf{123}$

Worked — $x + 1/x = 4$, find $x^3 + 1/x^3$

Direct — $k^3 - 3k = 4^3 - 3\times4 = 64 - 12 = \mathbf{52}$.

Check via steps — $x^2+1/x^2 = 16-2 = 14$; then $(x+1/x)(x^2+1/x^2-1) = 4\times13 = 52$ ✓. ✅

✓ Two-second recall ⟶ square gives $\mathbf{k^2-2}$, cube gives $\mathbf{k^3-3k}$. Just remember "minus 2, minus 3k".

The $x - 1/x$ Family (the signs flip)

If $x - 1/x = k$ (the "minus" family)

- $x^2 + 1/x^2 = k^2 + 2$ (note the +2 now)
- $x^3 - 1/x^3 = k^3 + 3k$
- $x^4 + 1/x^4 = (k^2+2)^2 - 2$

Bridge between families → $x^2+1/x^2 = (x+1/x)^2 - 2 = (x-1/x)^2 + 2$, and $(x+1/x)^2 - (x-1/x)^2 = 4$.

Worked — $x - 1/x = 3$, find x^2+1/x^2 and x^3-1/x^3

Square part — $k^2 + 2 = 9 + 2 =$ **11**.

Cube part — $k^3 + 3k = 27 + 9 =$ **36**. ✅

Worked — $x^2 + 1/x^2 = 7$, find $x + 1/x$ and $x - 1/x$

Plus — $(x+1/x)^2 = 7 + 2 = 9 \to x + 1/x =$ **± 3**.

Minus — $(x-1/x)^2 = 7 - 2 = 5 \to x - 1/x =$ **$\pm\sqrt{5}$**. ✅

A common twist — when x = the value itself

If $x^2 - 3x + 1 = 0$ $(x \neq 0)$, divide by $x \to x + 1/x = 3$. $\therefore$ $x^3+1/x^3 = 27-9 =$ **18**. Dividing the equation by x is the key first move.

Try these

$x+1/x=5 \to x^2+1/x^2 =$ **23**

$x+1/x=5 \to x^3+1/x^3 = 125-15 =$ **110**

$x-1/x=2 \to x^2+1/x^2 =$ **6**

$x+1/x=2 \to x = 1 \to x^n+1/x^n =$ **2**

$x + 1/x$ — Exam Variations

Same engine, disguised inputs. Learn to spot the hidden $x + 1/x$ and reduce.

Worked — if $x = 2 + \sqrt{3}$, find $x + 1/x$

Step 1 — $1/x = 1/(2+\sqrt{3}) = 2 - \sqrt{3}$ (rationalise; $(2+\sqrt{3})(2-\sqrt{3})=1$).

Step 2 — $x + 1/x = (2+\sqrt{3}) + (2-\sqrt{3}) = \mathbf{4}$.

Then — $x^3+1/x^3 = 4^3-3\times4 = \mathbf{52}$. ✅

Worked — if $x + 1/x = \sqrt{3}$, find $x^{18} + x^{12} + x^6 + 1$

Step 1 — square: $x^2+1/x^2 = 3-2 = 1 \to x^2-x+1\ldots$ better: $x^2+1 = \sqrt{3}\cdot x$.

Step 2 — cube relation gives $x^3+1/x^3 = (\sqrt{3})^3-3\sqrt{3} = 3\sqrt{3}-3\sqrt{3} = 0 \therefore x^6 = -1$.

Step 3 — $x^{18}+x^{12}+x^6+1 = (-1)^3+(-1)^2+(-1)+1 = -1+1-1+1 = \mathbf{0}$. ✅

✓ When $x + 1/x = 1 \longrightarrow x^3+1/x^3 = 1-3 = -2$; when $= \sqrt{3} \longrightarrow$ cube $= 0$; when $= \sqrt{2} \longrightarrow$ cube $= 2\sqrt{2}-3\sqrt{2} = -\sqrt{2}$.

Worked — if $x^2 + 1 = 4x$, find $x^2 + 1/x^2$

Step 1 — divide by x: $x + 1/x = 4$.

Step 2 — $x^2+1/x^2 = 4^2 - 2 = \mathbf{14}$. ✅

Try these

$x=3+2\sqrt{2} \to x+1/x = \mathbf{6}$

$x+1/x=1 \to x^3+1/x^3 = \mathbf{-2}$

$x^2-5x+1=0 \to x+1/x = \mathbf{5}$

$x-1/x=4 \to x^3-1/x^3 = 64+12 = \mathbf{76}$

⑤ Componendo & Dividendo

If $a/b = c/d$ then $(a+b)/(a-b) = (c+d)/(c-d)$

add 1 & subtract 1 on both sides, then divide — used to collapse messy ratios fast.

Worked – if $x/y = 7/3$, find $(x+y)/(x-y)$

Apply — $(x+y)/(x-y) = (7+3)/(7-3) = 10/4 = \mathbf{5/2}$. ✅

Worked – solve $(a+b)/(a-b) = 5/3$

Reverse C&D — $a/b = (5+3)/(5-3) = 8/2 = \mathbf{4}$ ∴ $a : b = 4 : 1$. ✅

Whenever you see (something + something)/(same − same) equal to a ratio, reach for Componendo & Dividendo – it removes the fractions instantly.

⑥ Remainder & Factor Theorem

- **Remainder Theorem** → when $P(x)$ is divided by $(x - a)$, the remainder = $\mathbf{P(a)}$.
- **Factor Theorem** → $(x - a)$ is a factor of $P(x) \Leftrightarrow \mathbf{P(a) = 0}$.

For $(x + a)$ use $P(-a)$; for $(bx - a)$ use $P(a/b)$.

Worked – remainder when $P(x) = x^3 - 2x^2 + 3x - 5$ is divided by $(x - 2)$

Put x = 2 — $P(2) = 8 - 8 + 6 - 5 = \mathbf{1}$ → remainder = 1. ✅

Worked – is $(x - 1)$ a factor of $x^3 - 6x^2 + 11x - 6$?

Put x = 1 — $1 - 6 + 11 - 6 = \mathbf{0}$ ∴ **yes**, $(x-1)$ is a factor. ✅

⑦ Quadratic Equations

Standard form $ax^2 + bx + c = 0$ ($a \neq 0$). Everything below flows from the roots α, β.

Core formulas

- Roots: $x = [\,-b \pm \sqrt{(b^2 - 4ac)}\,] / 2a$
- Sum of roots: $\alpha + \beta = -b/a$ • Product: $\alpha\beta = c/a$
- Discriminant: $D = b^2 - 4ac$
- Form an equation: $x^2 - (\text{sum})x + (\text{product}) = 0$

Nature of roots from D

Discriminant D	Nature of roots
$D > 0$	real & **distinct** (two different real roots)
$D = 0$	real & **equal** (coincident, $x = -b/2a$)
$D < 0$	**imaginary** / complex (conjugate pair)

If D is a perfect square (and a,b,c rational) → roots are rational; else irrational conjugates $p \pm \sqrt{q}$.

Worked — solve $x^2 - 5x + 6 = 0$

D = 25 − 24 = 1 (>0, perfect square) → rational distinct.

Roots = $(5 \pm 1)/2 = 3, 2$. Check: sum $5 = -(-5)$, product $6 = c/a$. **x = 2, 3.** ✅

Worked — for which k does $x^2 - kx + 9 = 0$ have equal roots?

D = 0 → $k^2 - 36 = 0$ → $k = \mathbf{\pm 6}$. ✅

Roots: Symmetric Functions & Forming Equations

Handy symmetric results (with $s = \alpha+\beta$, $p = \alpha\beta$)

- $\alpha^2 + \beta^2 = s^2 - 2p$
- $\alpha^2 - \beta^2 = s\sqrt{(s^2-4p)}$
- $(\alpha - \beta)^2 = s^2 - 4p$
- $1/\alpha + 1/\beta = s/p$
- $\alpha^3 + \beta^3 = s^3 - 3ps$

Worked — roots of $x^2 - 7x + 5 = 0$; find $\alpha^2 + \beta^2$

Read off — $s = 7$, $p = 5$.

Apply — $\alpha^2+\beta^2 = s^2 - 2p = 49 - 10 =$ **39**. ✅

Worked — form the equation whose roots are 3 and −5

Sum $= 3 + (-5) = -2$, **Product** $= 3\times(-5) = -15$.

Equation → $x^2 - (-2)x + (-15) =$ **$x^2 + 2x - 15 = 0$**. ✅

If one root is $p + \sqrt{q}$ (irrational, rational coeffs), the other MUST be its conjugate $p - \sqrt{q}$. Same for complex $a + bi \longrightarrow a - bi$.

⑧ Max & Min of a Quadratic

For $f(x) = ax^2 + bx + c$, the vertex is at $x =$ **$-b/2a$**, giving value **$(4ac - b^2)/4a$**. If $a > 0 \rightarrow$ **minimum**; if $a < 0 \rightarrow$ **maximum**.

Worked — minimum of $x^2 - 6x + 11$

Vertex $x = 6/2 = 3$; value $= (4\cdot 11 - 36)/4 = 8/4 =$ **2** ($a=1>0 \rightarrow$ minimum). ✅

⑨ Linear Equations — Consistency

For a pair $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, compare the coefficient ratios.

Condition	Solution / Graph
$a_1/a_2 \neq b_1/b_2$	**unique** solution (lines intersect) — consistent
$a_1/a_2 = b_1/b_2 = c_1/c_2$	**infinite** solutions (same line) — consistent, dependent
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	**no** solution (parallel lines) — inconsistent

Worked — classify $2x + 3y = 5$ & $4x + 6y = 10$

$2/4 = 3/6 = 5/10 = ½$ → all equal ∴ **infinite solutions** (same line). Change RHS to 9 → last ratio $5/9$ differs → **no solution**. ✅

Worked — find k so $kx + 2y = 5$ & $3x + y = 1$ have a unique solution

Need $a_1/a_2 \neq b_1/b_2$ → $k/3 \neq 2/1$ → $\mathbf{k \neq 6}$. ✅

⑩ Indices & Surds Recap

- $a^m \cdot a^n = a^{m+n}$
- $a^m / a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{-n} = 1/a^n$
- $a^{1/n} = \sqrt[n]{a}$
- $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

Rationalise: $1/(\sqrt{a}+\sqrt{b}) \times (\sqrt{a}-\sqrt{b})/(\sqrt{a}-\sqrt{b}) = (\sqrt{a}-\sqrt{b})/(a-b)$.

Worked — simplify $(2^5 \times 2^3)/2^6$

$= 2^{5+3-6} = 2^2 = \mathbf{4}$. Also $9^{1/2} = 3$, $27^{1/3} = 3$, $16^{-1/2} = ¼$. ✅

⑪ Symmetric-Expression Tricks

High-frequency SSC results

- If $a^2+b^2+c^2 = ab+bc+ca \rightarrow \mathbf{a = b = c}$.
- $a^2+b^2+c^2 - ab-bc-ca = ½[(a-b)^2+(b-c)^2+(c-a)^2] \geq 0$.
- If $a+b+c = 0 \rightarrow a^3+b^3+c^3 = 3abc$.
- $a^4+a^2+1 = (a^2+a+1)(a^2-a+1)$.

Why a=b=c? Because $a^2+b^2+c^2-ab-bc-ca = 0$ forces each squared bracket to $0 \longrightarrow a-b = b-c = c-a = 0$.

Worked – if $a+b+c = 0$, find $(a^2+b^2+c^2)/(ab+bc+ca)$

$a^2+b^2+c^2 = -2(ab+bc+ca)$ ∴ ratio = **-2**. ✅

Worked – if $a/b + b/a = 1$, find $a^3 + b^3$

$a/b + b/a = 1 \rightarrow a^2+b^2 = ab \rightarrow a^2-ab+b^2 = 0$.

$a^3+b^3 = (a+b)(a^2-ab+b^2) = (a+b)\times 0 =$ **0**. ✅

Try these

$a^2+b^2+c^2=ab+bc+ca \rightarrow a:b:c =$ **1:1:1**

$a+b+c=0 \rightarrow a^3+b^3+c^3 =$ **$3abc$**

$x-y=2,\ x^3-y^3=?$ need xy — **$8+6xy$**

$a/b+b/a=2 \rightarrow a=b \rightarrow a^3+b^3=$**$2a^3$**

✓ *Sum-of-squares = product-of-pairs is a* ***hidden equality flag****: the moment you see it, write a = b = c and finish.*

Algebra Formula Recap

Identities

Expression	Equals
$(a\pm b)^2$	$a^2 \pm 2ab + b^2$
a^2-b^2	$(a+b)(a-b)$
$(a\pm b)^3$	$a^3 \pm b^3 \pm 3ab(a\pm b)$
a^3+b^3	$(a+b)(a^2-ab+b^2)$
a^3-b^3	$(a-b)(a^2+ab+b^2)$
$(a+b+c)^2$	$a^2+b^2+c^2+2(ab+bc+ca)$
$a^3+b^3+c^3-3abc$	$(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
$a+b+c = 0 \Rightarrow$	$a^3+b^3+c^3 = 3abc$

The $x + 1/x$ results

Given	x^2+1/x^2	cube	x^4+1/x^4
$x+1/x = k$	k^2-2	k^3-3k	$(k^2-2)^2-2$
$x-1/x = k$	k^2+2	k^3+3k	$(k^2+2)^2-2$

Quadratic $ax^2+bx+c = 0$

Quantity	Formula
Roots	$[-b \pm \sqrt{(b^2-4ac)}]/2a$
Sum / Product	$\alpha+\beta = -b/a$ · $\alpha\beta = c/a$
Discriminant	$D = b^2-4ac$ ($>0, =0, <0$)
Build equation	$x^2-(\text{sum})x+(\text{product})=0$

Componendo-Dividendo: $a/b=c/d \Rightarrow (a+b)/(a-b)=(c+d)/(c-d)$ · Remainder = $P(a)$ · Factor $\Leftrightarrow$ $P(a)=0$.

Solved Examples 3

Q1. If $x + 1/x = 5$, find $x^2 + 1/x^2$ and $x^3 + 1/x^3$.

$x^2+1/x^2 = 5^2-2 =$ **23**; $x^3+1/x^3 = 5^3-3\times5 = 125-15 =$ **110**.

Q2. If $a + b = 7$ and $ab = 10$, find $a^3 + b^3$.

$a^3+b^3 = (a+b)^3 - 3ab(a+b) = 343 - 3\times10\times7 = 343 - 210 =$ **133**.

Q3. If $a + b + c = 0$, evaluate $(a^3+b^3+c^3)/abc$.

$a+b+c = 0 \Rightarrow a^3+b^3+c^3 = 3abc \therefore$ ratio $=$ **3**.

Q4. Find the remainder when $x^3 + 3x^2 - 4x + 2$ is divided by $(x + 1)$.

Put $x = -1$: $(-1) + 3 + 4 + 2 =$ **8**.

Q5. The roots of $2x^2 - 9x + 4 = 0$ are α, β. Find $\alpha + \beta$ and $\alpha\beta$.

$\alpha+\beta = -(-9)/2 =$ **9/2**; $\alpha\beta = 4/2 =$ **2**. (roots 4 & ½ ✓)

Q6. If $x = 3 + 2\sqrt{2}$, find $x^2 + 1/x^2$.

$1/x = 3 - 2\sqrt{2} \Rightarrow x + 1/x = 6 \therefore x^2+1/x^2 = 6^2-2 =$ **34**.

✓ *Pattern across all six ⟶ convert the given data into **sum, difference or product**, then drop it into the matching identity. Never solve for the variables themselves.*

Practice Set

1. $(a+b)^2$ if a+b... expand $(x+6)^2 = ?$
2. $67^2 - 33^2 = ?$
3. $a+b=8,\ ab=15 \rightarrow a^2+b^2 = ?$
4. $a+b=6,\ ab=8 \rightarrow a^3+b^3 = ?$
5. $x + 1/x = 4 \rightarrow x^2+1/x^2 = ?$
6. $x + 1/x = 3 \rightarrow x^3+1/x^3 = ?$
7. $x - 1/x = 2 \rightarrow x^2+1/x^2 = ?$
8. $a+b+c=0,\ abc=4 \rightarrow a^3+b^3+c^3 = ?$
9. $x/y = 5/2 \rightarrow (x+y)/(x-y) = ?$
10. remainder of $x^2-5x+7 \div (x-3) = ?$
11. D & nature: $x^2-4x+4 = 0 = ?$
12. roots sum/product of $3x^2-12x+9 = ?$
13. min value of $x^2-8x+20 = ?$
14. eqn with roots 2, 7 = ?

Answer Key — with working

1→ $x^2+12x+36$ · **2→ 3400** (100×34) · **3→ 34** (64−30) · **4→ 72** (216−3·8·6) · **5→ 14** (16−2) · **6→ 18** (27−9) · **7→ 6** (4+2) · **8→ 12** (3abc=3×4) · **9→ 7/3** ((5+2)/(5−2)) · **10→ 1** (P(3)=9−15+7) · **11→ D=0, equal** (x=2) · **12→ sum 4, prod 3** · **13→ 4** (vertex x=4) · **14→ $x^2-9x+14=0$**.

Quick Revision

✓ Square: $(a\pm b)^2$ · $a^2-b^2=(a+b)(a-b)$. Cube: $a^3\pm b^3$ factors & $(a\pm b)^3$.

✓ $x+1/x=k \rightarrow k^2-2,\ k^3-3k$; $x-1/x=k \rightarrow k^2+2,\ k^3+3k$. $a+b+c=0 \rightarrow \Sigma a^3=3abc$.

✓ Quadratic: $\alpha+\beta=-b/a$, $\alpha\beta=c/a$, $D=b^2-4ac$. Remainder=P(a); factor⇔P(a)=0.

☆ Identities & roots — algebra unlocked! ☆